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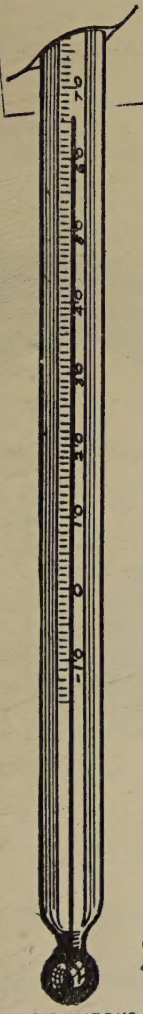

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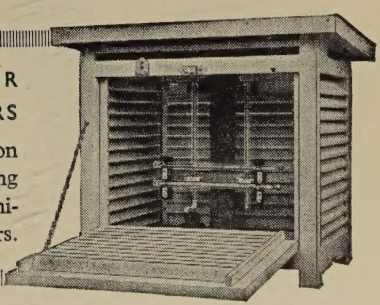

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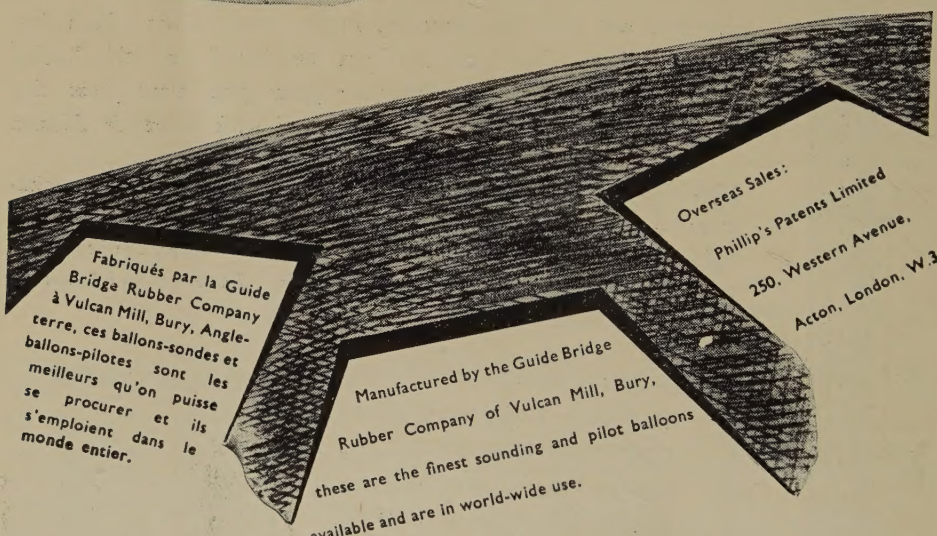

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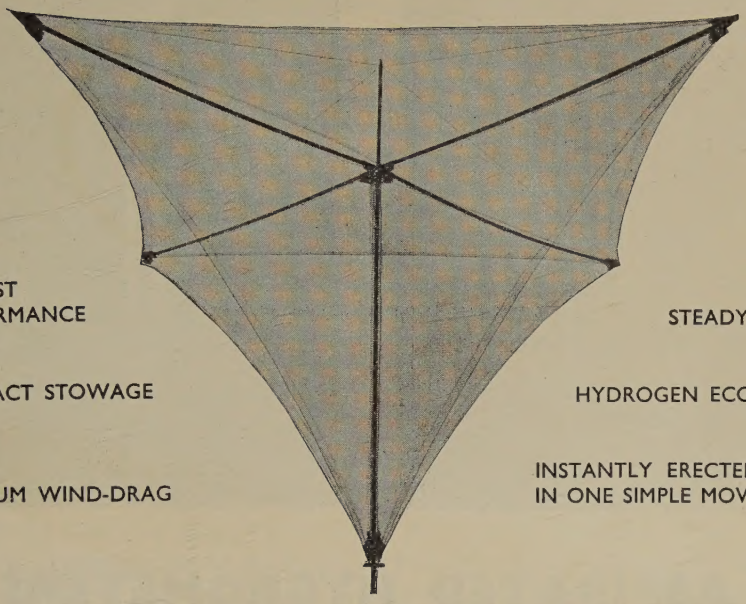

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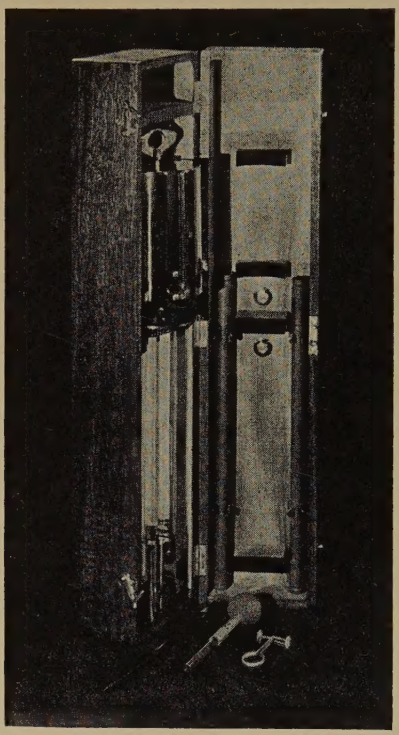

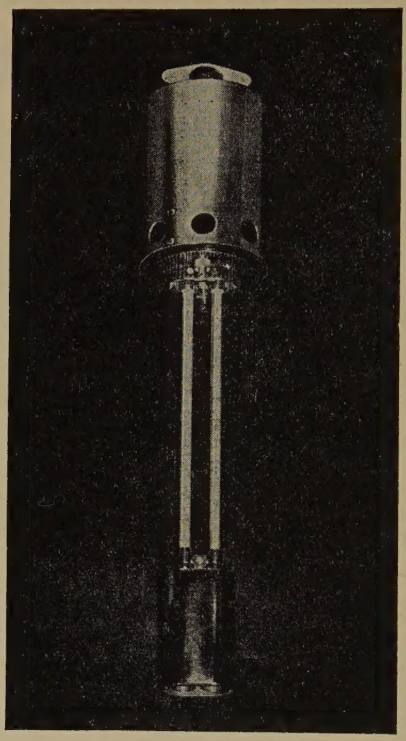

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Tidal oscillations of the lower stratosphere

By D. H. JOHNSON
Meteorological Office, Dunstable

(Read 16 July 1954 at the Edinburgh Meeting, Symposium on the Upper Atmosphere)

SUMMARY

It is demonstrated that diurnal and semi-diurnal variations of wind occur in the lower stratosphere. These oscillations appear to be associated with the solar tides. The stratospheric semi-diurnal oscillation of wind is in phase with, and of comparable magnitude to, the surface semi-diurnal oscillation of wind. In the stratosphere, the diurnal oscillation is found to be of similar magnitude to the semi-diurnal oscillation.

1. INTRODUCTION

Whilst methods of representing the 100-mb wind field were under investigation, a diurnal variation was detected in the winds observed at the 100-mb level. A study of the periodic variations of wind in the lower stratosphere has been made in order to establish their significance, form and magnitude.

2. REDUCTION OF DATA

Occasions in 1950-51 were selected when winds were reported at 100 mb from a British upper-air station at times t hr and $(t + 6)$ hr on the same day. Four sets of data were extracted, corresponding to the four 6-hr intervals 09 GMT–03 GMT, 15 GMT–09 GMT, 21 GMT–15 GMT and 03 GMT–21 GMT. The observed winds were resolved into south and west components and two-year monthly mean components of the wind change over the 6-hr interval were computed for each period. These are reproduced in Table 1.

TABLE 1. THE OBSERVED MONTHLY MEAN 6-HR CHANGES OF WIND (KT) AT 100 MB OVER THE BRITISH ISLES FOR 1950-51 (Number of cases given, in brackets, under each entry)

Component	Interval GMT	J	F	M	A	M	J	J	A	S	O	N	D	Annual Mean
West	09-03	−0·85 (111)	−1·20 (90)	−1·25 (106)	−1·17 (110)	−1·42 (150)	−1·42 (151)	−2·40 (143)	−0·91 (120)	−0·73 (121)	−0·93 (151)	−1·77 (87)	−0·36 (90)	−1·23 (1,430)
	15-09	0·48 (134)	0·85 (119)	1·96 (169)	1·17 (166)	1·51 (212)	1·50 (200)	2·30 (185)	1·93 (175)	1·52 (146)	1·90 (180)	2·00 (104)	0·02 (107)	1·50 (1,897)
	21-15	0·16 (123)	−0·42 (97)	0·04 (121)	−0·56 (103)	0·50 (134)	−0·41 (141)	−1·09 (129)	−0·11 (134)	0·07 (120)	−0·92 (165)	−0·67 (92)	−0·44 (120)	−0·32 (1,479)
	03-21	0·58 (128)	0·13 (77)	0·32 (100)	0·47 (92)	0·15 (109)	0·43 (141)	1·14 (120)	0·46 (107)	0·30 (116)	0·51 (165)	1·17 (84)	0·62 (101)	0·52 (1,340)
South	09-03	−1·58 (111)	−0·81 (90)	0·07 (106)	−0·10 (110)	0·26 (150)	−0·07 (151)	−0·22 (143)	−0·03 (120)	−0·90 (121)	0·25 (151)	−0·33 (87)	0·10 (90)	−0·24 (1,430)
	15-09	2·38 (134)	2·03 (119)	1·30 (169)	1·42 (166)	0·76 (212)	0·62 (200)	1·64 (185)	0·88 (175)	2·43 (146)	0·83 (180)	0·36 (104)	−0·16 (107)	1·23 (1,897)
	21-15	−0·94 (123)	−3·48 (97)	−2·04 (121)	−1·08 (103)	−1·70 (134)	−1·65 (141)	−2·01 (129)	−1·96 (134)	−1·99 (120)	−1·16 (165)	−1·88 (92)	−0·23 (120)	−1·64 (1,479)
	03-21	1·95 (128)	1·33 (77)	0·85 (100)	1·09 (92)	0·32 (109)	0·78 (141)	0·37 (120)	−0·73 (107)	0·59 (116)	0·71 (165)	1·21 (84)	0·14 (101)	0·72 (1,340)

No very critical statistical test is required to show that at least some of the annual means given in Table 1 differ significantly from zero, e.g. for the 09 GMT–03 GMT west component variation, all the monthly means have the same sign. If the monthly means themselves belonged to a population whose mean was zero, the odds against their having the same sign would be $(2^{11} - 1) : 1$, i.e. 2047 : 1. The magnitudes of the annual mean 6-hr variations are less than the standard error (3 or 4 kt) of the individual wind observations and are also less than the 6-hr synoptic variations. To obtain some indication of the general level of accuracy of the individual monthly means, an estimate has been made of the standard errors of the 15 GMT–09 GMT variations for four months. The individual values of the 15 GMT–09 GMT variations do not form an independent set because, although the serial correlation for individual stations is negligible, a small but significant correlation does exist between simultaneous values at different stations. If the data were composed of simultaneous values from k stations and the average correlation co-efficient between pairs of stations was $\bar{\rho}$, it can be shown that the standard error of the mean of n observations is given by

$$\sigma_m = \sigma \left[\{1 + (k - 1)\,\bar{\rho}\}/n\right]^{\frac{1}{2}},$$

where σ is the standard deviation of the population from which the n observations are drawn. This replaces the usual relation for independent values of $\sigma_m = \sigma/n^{\frac{1}{2}}$. Using the former relation with appropriate values of k and $\bar{\rho}$ the standard errors of the monthly means in June, July, December and January respectively were estimated to be, for the south component, 0·43 kt, 0·53 kt, 1·00 kt and 0·70 kt; and for the west component, 0·39 kt, 0·42 kt, 0·65 kt and 0·54 kt. Evidently a number of the monthly means themselves differ significantly from zero.

If a_1, a_2, a_3 and a_4 are the four true 6-hr variations of wind then, neglecting the seasonal variation of wind, which is small in comparison with the diurnal changes, we can put $a_1 + a_2 + a_3 + a_4 = 0$. This relationship is not obeyed exactly by the variations given in Table 1 because, on many days, winds are lacking at one or more of the hours of observation. For these days, only one or two of the 6-hr variations were obtained. We can thus obtain from Table 1 two estimates of each of the a_i; one is given directly by a_i, and the other is given by $-(a_j + a_k + a_l)$. The means of the two estimates form the basis of the analysis which follows. They are illustrated graphically in Fig. 1.

Inspection of Fig. 1 suggests that any seasonal variations which may occur in the diurnal changes are smaller in magnitude than the annual means. Average 6-hr changes were calculated for the winter (November to March) and summer half-years. These are given in Table 2. Student's t-test applied to these data showed that the differences between the winter and summer means were in no case significant, even at the 10 per cent level, suggesting that seasonal trends are small in comparison with chance sampling fluctuations.

TABLE 2. MEAN 6-HR VARIATIONS OF 100-MB WIND FOR WINTER AND SUMMER HALF-YEARS, 1950-51

Interval GMT	West component (kt)		South component (kt)	
	Summer	Winter	Summer	Winter
09-03	− 1·61	− 1·22	− 0·09	− 0·45
15-09	1·39	1·04	1·40	1·05
21-15	− 0·53	− 0·53	− 1·62	− 1·69
03-21	0·09	0·39	0·51	0·95

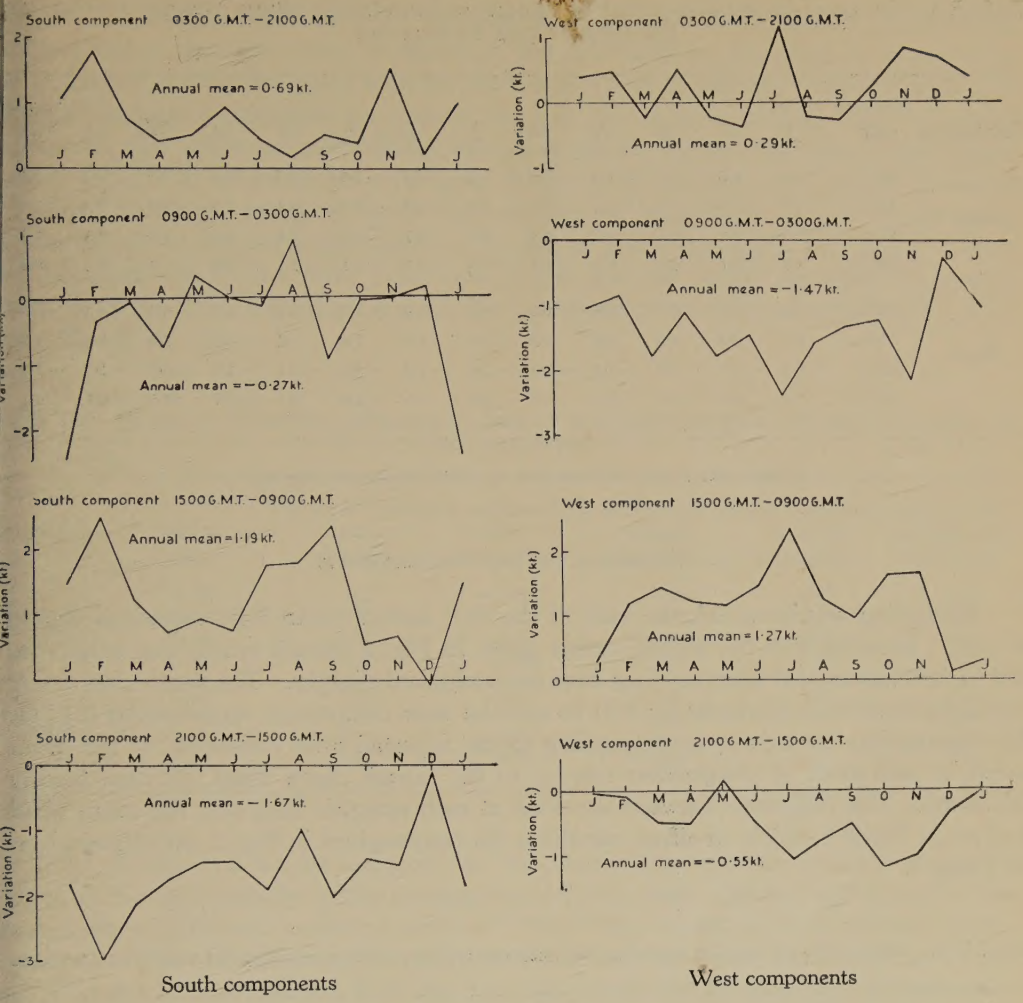

South component 0300 G.M.T. – 2100 G.M.T.
Annual mean = 0·69 kt.
West component 0300 G.M.T. – 2100 G.M.T.
Annual mean = 0·29 kt.
South component 0900 G.M.T. – 0300 G.M.T.
Annual mean = – 0·27 kt.
West component 0900 G.M.T. – 0300 G.M.T.
Annual mean = – 1·47 kt.
South component 1500 G.M.T. – 0900 G.M.T.
Annual mean = 1·19 kt.
West component 1500 G.M.T. – 0900 G.M.T.
Annual mean = 1·27 kt.
South component 2100 G.M.T. – 1500 G.M.T.
Annual mean = – 1·67 kt.
West component 2100 G.M.T. – 1500 G.M.T.
Annual mean = – 0·55 kt.
Variation (kt.)
J F M A M J J A S O N D J
South components
West components

Figure 1. Monthly mean 6-hr variations of 100-mb wind, 1950-51.

A test was made to determine whether the differences between any of the monthly and annual mean 6-hr variations were so great as to merit special investigation. The variances of the monthly means about the annual means averaged 0·26 kt for the west components and 0·46 kt for the south components. None of the west component monthly means differed significantly from the annual mean. For the south component, the 09 GMT – 03 GMT January monthly mean differed from the corresponding annual mean by an amount, 2·16 kt, equal to 3·19 times the standard deviation. The odds against such a difference occurring by chance on any specified occasion are 998·56 : 1·44, i.e. 693 : 1 but the odds against such a difference occurring by chance in a sample of 48 values are $(\cdot 99856)^{48} : 1 - (\cdot 99856)^{48}$, i.e. 14 : 1. The deviation of the January value from the annual mean is therefore of doubtful significance.

In addition to the data described above, use has been made of some unpublished data made available, by H. W. L. Absalom, for study by the writer. These data consist of monthly and annual mean 6-hr variations of 150-mb wind. They are particularly valuable in that they refer to the years 1944-47 when upper-wind observations were made at 06 GMT, 12 GMT, 18 GMT and 24 GMT. They are given in Table 3.

TABLE 3. THE OBSERVED MONTHLY MEAN 6-HR CHANGES OF WIND (KT) AT 150-MB OVER THE BRITISH ISLES FOR 1944-47* (DUE TO ABSALOM)

Component	Interval GMT	J	F	M	A	M	J	J	A	S	O	N	D	Annual Mean
West	06-00	0·0	1·6	0·6	−1·6	0·5	−0·9	−0·5	−1·1	−2·5	0·0	−0·9	2·5	−0·2
	12-06	0·4	−0·8	−2·7	1·2	−0·9	0·1	1·3	−0·7	−0·4	−1·5	−0·5	−2·3	−0·6
	18-12	1·8	0·8	0·6	1·3	1·5	0·5	0·7	1·7	2·0	1·5	3·0	0·3	1·3
	00-18	1·7	−2·1	1·3	−1·7	−0·3	−1·5	−0·3	0·1	−1·1	−0·2	0·3	0·2	−0·3
South	06-00	−1·3	0·7	0·6	−0·5	1·0	0·7	−1·9	−1·3	−0·7	0·1	1·3	−2·3	−0·3
	12-06	1·7	0·3	0·8	2·2	1·5	1·7	2·5	1·8	0·2	0·6	3·8	3·6	1·7
	18-12	−0·2	−1·5	−1·3	−1·3	−1·5	−1·8	−1·4	−1·1	−0·1	−1·9	−2·2	−1·4	−1·3
	00-18	3·1	0·8	−1·1	0·5	0·6	−1·4	0·1	−1·0	1·1	−3·4	4·6	−0·1	0·3

* The 00 GMT-18 GMT variations were calculated from 2 years' data only.

3. ANALYSIS OF THE OSCILLATIONS

Neglecting secular trend, the sum of the four annual mean 6-hr variations should be zero. In order that the annual means given in Fig. 1 should satisfy this condition, and to facilitate further analysis, they have been adjusted slightly. The south component variations have been increased by 0·01 kt and the west component variations by 0·12 kt. The appropriate deviations, corresponding to the adjusted 6-hr variations, of the mean winds at each hour of observation relative to the overall mean wind are then readily calculated. The difference between the wind at each synoptic hour and the mean wind is given in Table 4 which, in effect, combines the results given in Fig. 1 with those given in Table 3.

TABLE 4. DEVIATION OF ANNUAL MEAN WIND-COMPONENTS FOR STATED HOURS FROM THE OVERALL MEANS

Time GMT	03	06	09	12	15	18	21	24
South component (kt)	0·01	−0·54	−0·25	1·03	0·95	−0·40	−0·71	−0·11
West component (kt)	0·43	−0·05	−0·92	−0·70	0·46	0·55	0·03	0·20

The compound oscillations represented by the data in Table 4 have been broken down by harmonic analysis. For each wind component it is possible to obtain four harmonics, given by the equations:

$$100\ S = -54\cos(\theta + 32°14') + 67\cos(2\theta + 44°24') - 6\cos(3\theta + 80°32') + 0{\cdot}25\cos 4\theta; \quad (1)$$

$$100\ W = 51\cos(\theta + 84°24') + 51\cos(2\theta - 29°3') - 7\cos(3\theta - 26°34') + 0{\cdot}0\cos 4\theta; \quad (2)$$

where S and W are the south-wind and west-wind components in kt, and θ varies from 0 to 2π as time varies through 24 hr from 03 GMT.

Eqs. (1) and (2) reveal that the amplitudes of the first two harmonics considerably xceed those of the latter two. A statistical test was made to determine to what extent ie individual harmonics could account for the observed variations. Table 5 gives the alues of Student's t obtained for each of the 6-hr variations treated in Fig. 1, in order examine the following possibilities, viz., that the twelve monthly means are drawn om a population whose mean is (*a*) zero, (*b*) given by the first harmonic, (*c*) given by ie second harmonic, (*d*) given by the first and second harmonics combined or (*e*) given y the sum of the first three harmonics. For each of these tests values of t, corresponding probabilities of 0·1 per cent, 1 per cent, 5 per cent and 10 per cent that the sample ested is drawn from a population with the postulated mean value, are 4·44, 3·11, 2·20 nd 1·80 respectively.

TABLE 5. VALUES OF STUDENT'S t OBTAINED IN TESTS OF THE SIGNIFICANCE OF THE OBSERVED SYSTEM OF 6-HR VARIATIONS

Component	Period GMT	Values of Student's t obtained for difference between observed variation and: zero variation	first harmonic	second harmonic	first two harmonics	second two harmonics
South	09-03	− 1·13	− 4·25	2·87	− 0·25	− 0·04
	15-09	5·59	4·82	1·23	0·45	0·18
	21-15	8·74	− 4·79	− 3·68	0·26	− 0·05
	03-21	4·87	6·00	− 1·53	− 0·40	0·07
North	09-03	− 8·29	− 5·00	− 3·00	0·29	− 0·23
	15-09	7·12	4·41	1·82	− 0·88	− 1·06
	21-15	− 4·42	− 9·08	3·08	− 1·58	− 0·83
	03-21	1·60	4·67	− 4·40	− 1·33	− 1·13

From Table 5 it can be deduced that the observed system of variations cannot be ccounted for satisfactorily on the assumptions (*a*) that there is no real variation, (*b*) that hey consist of a diurnal oscillation alone or (*c*) that they consist of a semi-diurnal oscilla- ion alone. The variations can be explained satisfactorily by a combination of the diurnal nd semi-diurnal oscillations; it is not necessary to invoke harmonics of higher order han the second to account for the present data.

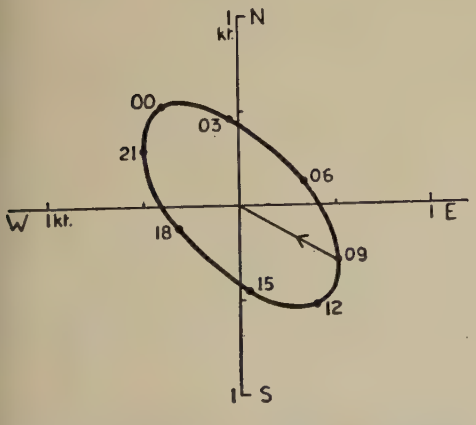


Figure 2. Vector diagram giving the wind velocity due to the diurnal oscillation. The wind velocity at any time is defined by the line joining the appropriate point on the ellipse to the origin.

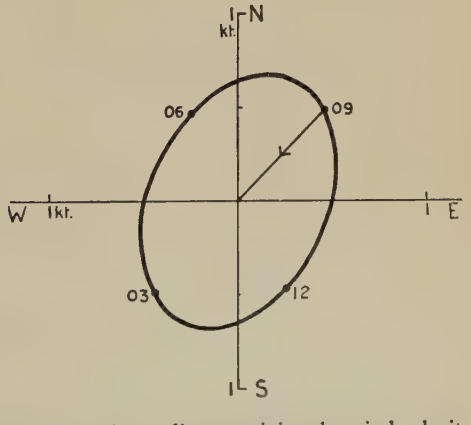
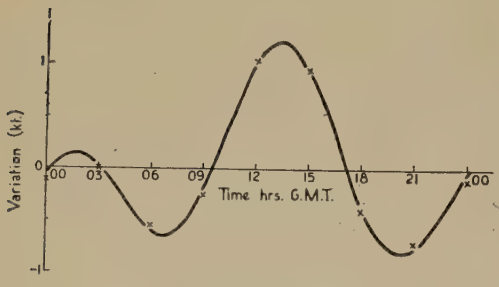


Figure 3. Vector diagram giving the wind velocity due to the semi-diurnal oscillation. The wind velocity at any time is defined by the line joining the appropriate point on the ellipse to the origin.

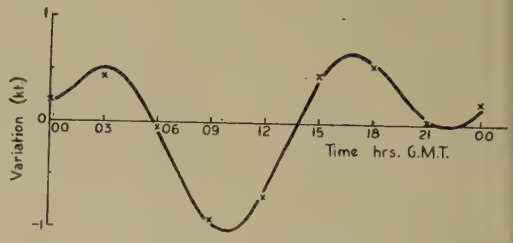
The diurnal oscillations of the south-wind and west-wind components are defined by the first terms on the right hand sides of Eqs. (1) and (2). These have been combined in Fig. 2, which is a vector diagram giving, for any time, the resultant wind velocity due to the diurnal oscillations. The semi-diurnal variation of wind, given by the second terms on the right-hand sides of Eqs. (1) and (2), is represented in a similar fashion in Fig. 3.

Fig. 4 shows the result of combining the diurnal and semi-diurnal oscillations and gives, for comparison, the observed variations.

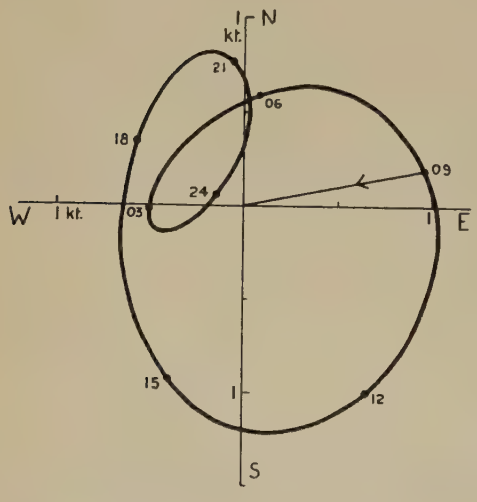
Fig. 5 depicts the kidney-shaped path which would be followed by particles moving solely with the wind system defined in Fig. 4 (c).



4 (a). South component.



4 (b). West component.



4 (c). Vector diagram.

Figure 4. The combined diurnal and semi-diurnal oscillations.
In (a) and (b) the crosses represent the observed deviation (given in Table 4) of the wind components from the mean. The curves give the sum of the first two harmonics of these observed variations. (c) The resultant wind velocity due to the combined diurnal and semi-diurnal oscillations. The wind velocity at any time is defined by the line joining the appropriate point on the curve to the origin.

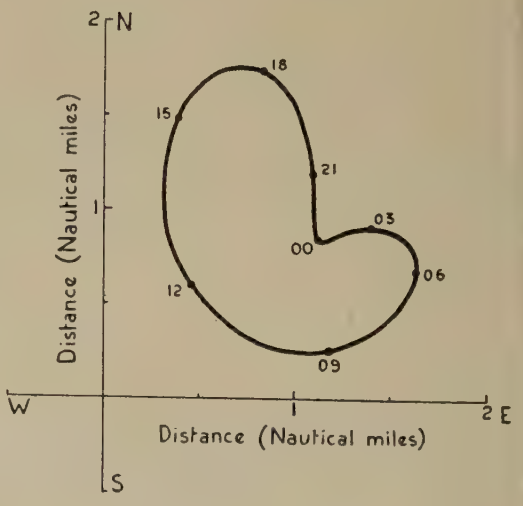


Figure 5. Path followed by particles oscillating only under the wind system defined by the combined diurnal and semi-diurnal variations.

4. Physical nature of the oscillations

In the preceding sections it has been established that the measured changes of wind the lower stratosphere contain small periodic components. Possible explanations of e periodic variations are : (i) that they are spurious and are due to some systematic ror of the wind measurements or (ii) that they are real and are associated with the al oscillations which are known to occur in the atmosphere.

It has not been possible to conceive of any direct instrumental effect which could count for the observed variations. The 1944-47 winds were measured for the most rt by radio direction-finding techniques, whereas the 1950-51 winds were measured the use of G.L. III radar equipment. It is difficult to envisage any inherent periodic strumental effects which could be common to both techniques.

Some diurnal effect could arise as follows. There is an apparent diurnal variation the measured height of the 100-mb surface. The apparent diurnal 15 GMT–03 GMT hange of contour height ranges from about 140 ft at the winter solstice to 380 ft at the mmer solstice, for stations in the British Isles. It is believed that this apparent variation height is largely false, and that it results from the effect of solar radiation on the radio-nde itself. It is likely that the 15 GMT '100-mb' winds refer to a level which is in ct from 140 ft to 380 ft higher than the true 100-mb surface. Consequently the wind bservations will contain a false diurnal variation, the magnitude of which depends on he shear of wind at 100 mb.

An examination of the shear of the mean wind in the lower stratosphere showed hat the shear in the south component is so small as to have a completely negligible effect t all seasons. At the most favourable time of the year (June and July), the shear could ive rise to an apparent 15 GMT–03 GMT variation in the west component of about 0·4 kt, ut from December to May the effect of shear in the west component is again negligible. nspection of Fig. 3 shows that the greatest 12-hr change of west wind takes place between bout 09 GMT and 21 GMT, a period when there is little apparent variation of contour eight. It is clear that for the annual means, we can discount the effect of the apparent iurnal variation of height of the 100-mb surface as being small in comparison with he observed diurnal variation of wind.

Atmospheric tides have been extensively studied in terms of the changes of pressure which occur at the earth's surface. The most important solar tides have periods of welve and twenty-four hours. Of these two, the semi-diurnal oscillation has attracted greater attention, partly because, at the earth's surface, it has the greater amplitude and shows greater uniformity than the diurnal oscillation. Simpson (1918) has given the following expression for the observed semi-diurnal pressure variation :

$$p = 0{\cdot}937 \cos^3 \theta \sin [2(\omega t + \lambda) + 154^\circ] \quad . \quad . \quad . \quad (3)$$

where p is pressure in mm of mercury, θ is latitude, ω is the earth's angular velocity, t is Greenwich mean time and λ is longitude east of Greenwich. Bartels (1928) has derived theoretically, the following expressions for the low-level wind components associated with the pressure variation given by Eq. (3) :

$$U = -54{\cdot}6 \sin \theta \cos [2(\omega t + \lambda) + 154^\circ]$$
$$V = -(21{\cdot}8 + 32{\cdot}8 \sin^2 \theta) \sin [2(\omega t + \lambda) + 154^\circ] \quad . \quad . \quad (4)$$

where U is the south component and V is the west component of wind in cm/sec.

It follows from expressions (3) and (4) that the pressure and west-wind component oscillations are in antiphase at the earth's surface, maximum west-wind component occurring at about 04 and 16 local time. In the northern hemisphere, the pressure and

south-wind component oscillations are in quadrature, maximum south-wind componen occurring at about 01 and 13 local time. Reference to Fig. 4 shows that the semi-diurna oscillation in the lower stratosphere is approximately in phase with the surface oscilla tion. This relationship is to be expected from tidal theory as given for example b Wilkes (1949).

The amplitudes of the surface tidal wind components given by (4) are, in latitud 55°N, 0·87 kt for the south component and 0·85 kt for the west component. These ma be compared with the lower stratosphere semi-diurnal amplitudes of 0·67 kt for th south component and 0·51 kt for the west component. It appears that there is no grea variation in the amplitude of the tidal winds, at least up to the 100 mb (16 km) level. I is interesting to note however that recent studies, by Elford and Robertson (1953) an others, of the drift of meteor trails, indicate that the semi-diurnal variations in the wind components have magnitudes of the order of 60 kt in the 80 km to 100 km layer.

As previously mentioned, the surface diurnal pressure-changes have not been studied to the same extent as the semi-diurnal pressure-changes. The present study indicates that the diurnal and semi-diurnal oscillations are of comparable magnitude in the lower stratosphere. Elford and Robertson's data show that the two effects are also of equal importance in the 80 km to 100 km layer.

5. Concluding remarks

There appears to be little doubt that the periodic variations of wind discussed in this paper are largely associated with the solar tides. Wind observations made at 100 mb are particularly suitable for study in connection with the tidal oscillations since, at least between latitudes 30°N and 60°N, the synoptic variations of wind are considerably smaller at that level than they are in the upper troposphere. The lower stratosphere is also free from the smaller-scale effects which would complicate studies of periodic wind variations occurring in the lower troposphere.

Acknowledgments

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The tilt of a 12-hourly atmospheric tidal wave

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SUMMARY

Forced oscillations of 12-hr period, stationary relative to the sun, are considered in a model atmosphere enclosed in a canal coincident with the earth's equator. Two-dimensional, inviscid motion is assumed. The study is based on equations of perturbation containing linear terms and second-order product terms. The oscillations are excited thermally and gravitationally. In the general case such waves have a variation of phase with height, a 'tilt,' and this is necessary in order that a vertical transport of energy may take place. In non-tilting waves no tidal energy can be supplied to the model atmosphere. An adiabatic wave can have no tilt. The method of perturbation is applicable to the tilting waves only for a limited time interval.

1. INTRODUCTION

The theory of tidal waves and related oscillations produced by the sun and the moon in the atmosphere has been investigated by many authors since the time of Laplace. In recent years, Taylor, Pekeris and Wilkes have examined tidal waves in a rotating spherical-model atmosphere, using the linearized hydrodynamical equations and assuming inviscid motion and adiabatic changes of state. In his latest paper Wilkes (1951) has also considered non-adiabatic, thermally excited oscillations. This was done simultaneously by the writer (Thrane 1951).

It is to be expected that the thermal excitation will be of special importance for the theory of the 12-hourly component of the solar barometric variation. It is the purpose of the present paper to call attention to a feature of this oscillation, which has received but little attention in theoretical work. Analysis of pressure records in the Alps, at elevations from 522 m to 4,575 m above sea level, has revealed a delay with height of the pressure extremes (Wagner 1932), the delay amounting to two hours at the 4,000-m level as compared to sea level. From the free atmosphere no pressure data of sufficient accuracy are available, from which it might be ascertained if the variation of phase exists there. Thus the question whether the findings of Wagner are related to purely local effects or to general properties of the global oscillation cannot be decided by observations, but possibly by theoretical considerations. J. Bjerknes (1948) has stressed the importance of Wagner's result, regarding it as an indication of a general phenomenon of global character. The present paper supports this view and calls attention to certain important properties of the wave connected with the variation of phase with height. These properties can be studied in the spherical-model atmosphere, but the essential features can also be brought out by using a simpler model, similar to that employed in the canal theory of the tides. For simplicity this method will be adopted in the following.

2. THE MODEL AND THE FUNDAMENTAL EQUATIONS

(a) *Choice of model and coordinate system*

We shall consider an atmosphere consisting of an ideal gas enclosed in a canal coincident with the earth's equator. In the static state the air is assumed to be at rest relative to the earth's surface. On this static state is superposed a forced oscillation having the following characteristics :

thermal and gravitational excitation due to the sun, assumed to be in its equinoctial position,

two-dimensional inviscid motion parallel to the equatorial plane,

small amplitude, 12-hr period,

wave velocity zero relative to the sun's meridian.

We shall further assume that no period of free oscillation is exactly 12-hr (although may be near to this value).

An individual particle of air in the equatorial plane will be identified by means of the Lagrangian coordinates, a, b, which, in the static state, coincide with the distance of the particle from the earth's axis and its longitude, respectively. Let pressure and specific volume be $p_0 = p_0(a)$ and $\alpha_0 = \alpha_0(a)$. In the state of oscillations the coordinates, pressure and specific volume of the particle will be denoted by

$$a + z, \quad b + \psi, \quad p_0 + p, \quad \alpha_0 + \alpha,$$

where z, ψ, p and α are functions of a, b and the time t.

Other symbols used in the following are :

c_p, c_v = specific heat at constant pressure, volume,

$\kappa = c_p/c_v$,

g = acceleration of gravity,

q = heat per unit mass entering the air,

$\epsilon = (\kappa - 1)\dfrac{\partial q}{\partial t}$,

Ω = angular velocity of the earth,

$\mu = b + \Omega t$,

$t_0 = \dfrac{\pi}{\Omega}$ = period of oscillation (12 hr),

$\rho_0 = 1/\alpha_0$ = density in the static state.

(b) *General equations; tide-producing potential*

Let R denote the distance of a particle M (a, b) from the earth's axis, Ψ the angle between a coordinate plane and the meridian plane of M, S and P its specific volume and pressure, and Φ the potential of the external force acting on M. The equations of motion in Lagrangian form may then be written (V. Bjerknes 1930, p. 28) :

$$(\ddot{R} - R\dot{\Psi}^2)\,\nabla R + \frac{\partial}{\partial t}(R^2\dot{\Psi})\,\nabla\Psi + S\nabla P + \nabla\Phi = 0, \qquad (1)$$

which represents two equations, as ∇ has to be replaced by $\dfrac{\partial}{\partial a}$ and $\dfrac{\partial}{\partial b}$ respectively. Let the coordinate system be chosen such that $\Psi = \pi$ is the meridian plane of the sun. Ψ is then local time reduced to angle. In the static state we have $\Psi = b + \Omega t$ and also $R = a$, $S = \alpha_0$, $P = p_0$. Further we may write

$$\Phi = \Phi_0(R) + \phi(R, \Psi),$$

where Φ_0 is the earth's gravitation potential and ϕ the tide-producing potential of the

un. If we postulate in the atmospheric model a potential of the form

$$\Phi_0 = gR + \tfrac{1}{2}\,\Omega^2 R^2, \quad . \quad . \quad . \quad . \quad (2)$$

valid in the vicinity of the earth) then equation (1) takes the form

$$[\ddot{R} - R(\dot{\Psi}^2 - \Omega^2)]\,\nabla R + \frac{\partial}{\partial t}(R^2\dot{\Psi})\,\nabla\Psi + S\nabla P + g\nabla R + \nabla\phi = 0, \quad . \quad (3)$$

which reduces to the hydrostatic equation $\alpha_0 p'_0 = -g$ in the static state.

The equation of continuity and the first law of thermodynamics may be written :

$$R\frac{D(R,\Psi)}{D(a,b)} = S\,a\,\rho_0, \quad . \quad . \quad . \quad . \quad (4)$$

$$\epsilon = (\kappa - 1)\frac{\partial q}{\partial t} = \kappa P\frac{\partial S}{\partial t} + S\frac{\partial P}{\partial t} \quad . \quad . \quad . \quad (5)$$

The tide-producing potential in the equatorial canal may be written

$$\phi = -g_1(R)\cos 2\Psi, \quad . \quad . \quad . \quad . \quad (6)$$

g_1 being a small positive quantity. In the first approximation we may here replace R by a and Ψ by $(b + \Omega t)$. Differentiation with respect to a will be indicated by a prime.

(c) *Equations of perturbation*

In Eqs. (3)–(5) we introduce $R = a + z$, $\Psi = b + \Omega t + \psi$, $S = \alpha_0 + \alpha$, $P = p_0 + p$. Assuming z, ψ, α, p and their derivatives to be small quantities, it is customary to simplify the equations by neglecting products of such quantities, whereby the linear equations of perturbation are obtained. However, certain important properties of the wave can only be studied by considering product-terms of the second order of magnitude. We shall therefore retain both first- and second-order terms in the equations, neglecting terms of the third and higher order.

The equations of motion, the equation of continuity and the first law of thermodynamics may then be written as follows :

$$\ddot{z} - 2\Omega\,a\,\dot{\psi} + \alpha_0\frac{\partial p}{\partial a} - g\,\rho_0\,\alpha + g\frac{\partial z}{\partial a} - g'_1\cos 2(b + \Omega t) = G_1, \quad . \quad (7)$$

$$a^2\ddot{\psi} + 2\Omega\,a\,\dot{z} + \alpha_0\frac{\partial p}{\partial b} + g\frac{\partial z}{\partial b} + 2g_1\sin 2(b + \Omega t) = G_2, \quad . \quad (8)$$

$$\frac{z}{a} + \frac{\partial z}{\partial a} + \frac{\partial\psi}{\partial b} - \rho_0\,\alpha = G_3, \quad . \quad (9)$$

$$\epsilon - \kappa\,p_0\,\dot{\alpha} - \alpha_0\,\dot{p} = G_4, \quad . \quad (10)$$

All linear terms are collected on the left-hand side, while $G_1 \ldots G_4$ are sums of product-terms of the second order.

The equations comprise the following cases :

Free oscillations (not treated here)	$(g_1 = 0,\ \epsilon = 0)$.
Forced oscillations :	
Gravitationally excited, adiabatic	$(g_1 \neq 0,\ \epsilon = 0)$,
thermally excited	$(g_1 = 0,\ \epsilon \neq 0)$,
combined gravitational-thermal	$(g_1 \neq 0,\ \epsilon \neq 0)$.

ϵ will be considered as a given quantity from a physical point of view, thus making it

unnecessary to refer to the laws of radiation and conduction of heat. This physical assumption does not preclude that the numerical value of ϵ may have to be determined by computation from directly observed quantities. ϵ will then have to be treated as " unknown " from a purely mathematical point of view.

We have four equations connecting 5 dependent variables (p, α, z, ψ, ϵ). At the outset the mathematical treatment will involve no distinction between known and unknown variables.

3. Wave solution

(a) *The first approximation; tilting waves*

Let us consider first the linearized equations obtained by putting $G_i = 0$ ($i = 1, 2, 3, 4$). For brevity we write $\mu = b + \Omega t$. In the classical theories complex expressions of the form $F(a)\, e^{in\mu}$ are substituted for the variables. In the very special case considered here the complex form no longer simplifies the formulae, and we shall use real functions only. It is immediately seen that the linear equations have a particular solution of the form

$$\psi_1 = A_\psi \sin 2\mu, \quad z_1 = A_z \cos 2\mu, \quad p_1 = A_p \cos 2\mu,$$
$$\epsilon_1 = A_\epsilon \sin 2\mu, \quad \alpha_1 = A_\alpha \cos 2\mu, \qquad . \quad . \quad . \quad . \quad (11)$$

where the amplitudes are functions of a.

In the case $g_1 = 0$ we replace A by B and 2μ by $2\mu + \eta_0$, (η_0 const.), thus obtaining :

$$\psi_1 = B_\psi \sin(2\mu + \eta_0) \ . \ . \ . \ . \ \alpha_1 = B_\alpha \cos(2\mu + \eta_0) \quad . \quad . \quad (12)$$

(11) is a gravitational-thermal oscillation while (12) is purely thermal. We note that the latter becomes a free oscillation if $B_\epsilon = 0$, and since we have assumed that no free oscillation of 12-hr period exists, we must have $B_\psi = B_p = B_z = B_\alpha = 0$ when $B_\epsilon = 0$.

Putting in (12) $\eta_0 = \frac{\pi}{2}$, and adding to (11), we obtain a new solution :

$$\psi_1 = A_\psi \sin 2\mu + B_\psi \cos 2\mu, \quad z_1 = A_z \cos 2\mu - B_z \sin 2\mu,$$
$$\epsilon_1 = A_\epsilon \sin 2\mu + B_\epsilon \cos 2\mu, \quad p_1 = A_p \cos 2\mu - B_p \sin 2\mu, \quad . \quad . \quad (13)$$
$$\alpha_1 = A_\alpha \cos 2\mu - B_\alpha \sin 2\mu.$$

(13) reduces to (12) in the special case when B_x/A_x has a constant value, the same for $x = \psi, \epsilon, z, p, \alpha$.

Writing in (13) :

$$z_1 = (A_z^2 + B_z^2)^{\frac{1}{2}} \cos(2\mu + \eta_z) \qquad . \quad . \quad . \quad (14)$$

we note that $\eta_z = \tan^{-1}(B_z/A_z)$ is a function of a. (13) and (14) will therefore be referred to as a ' tilting wave.'

Putting in Eqs. (7) to (10) $G_i = 0$ and introducing the expressions (13), we obtain for the amplitudes, 8 equations which may be written briefly as follows :

$$[-4\Omega^2 z - 4\Omega^2 a\, \psi + \alpha_0 p' - g\rho_0 \alpha + g z' - g_1']_{A,B} = 0, \quad . \quad . \quad (15)$$
$$[-2\Omega^2 a^2 \psi - (g + 2\Omega^2 a) z - \alpha_0 p + g_1]_{A,B} = 0, \quad . \quad . \quad (16)$$
$$\left[\frac{z}{a} + z' + 2\psi - \rho_0 \alpha\right]_{A,B} = 0, \quad . \quad . \quad (17)$$
$$[\epsilon + 2\Omega(\kappa p_0 \alpha + \alpha_0 p)]_{A,B} = 0, \quad . \quad . \quad (18)$$

where the subscripts A and B indicate that we have to replace z, ψ, α, p, ϵ by $A_z \ldots A_\epsilon$ and by $B_z \ldots B_\epsilon$ respectively. In the latter case g_1 and g'_1 have to be deleted, in accordance with the definition of B_x.

By suitable eliminations we may derive from Eqs. (15) to (18) the following equations:

$$A_z'' + K_1 A_z' + K_2 A_z = K_3 A_\epsilon' + K_4 A_\epsilon + K_5 g_1' + K_6 g_1, \quad . \quad (19)$$

$$B_z'' + K_1 B'_z + K_2 B_z = K_3 B_\epsilon' + K_4 B_\epsilon, \quad . \quad . \quad . \quad . \quad (20)$$

where the K_i are functions of a which may be computed from the coefficients of Eqs. (15) to (18).

Let us consider the case where the model atmosphere is enclosed between two rigid horizontal surfaces $a = a_0$ and $a = a_1$. The boundary conditions are $A_z = B_z = 0$ for $a = a_0$ and $a = a_1$. Let the heat supply per unit mass and unit time be given, i.e. A_ϵ and B_ϵ given. From the inhomogeneous differential equations (19) and (20) it is seen that A_z and B_z can be determined so as to satisfy the boundary conditions. The other amplitudes are then determined by Eqs. (15 to 18).

(b) *The second approximation*

The system (13) may be considered as the first approximation to a solution of the Eqs. (7) to (10). We shall seek a better approximation in the form

$$\psi = \psi_1 + \psi_2, \quad z = z_1 + z_2, \quad p = p_1 + p_2$$

$$\epsilon = \epsilon_1 + \epsilon_2, \quad \alpha = \alpha_1 + \alpha_2, \quad . \quad . \quad . \quad . \quad (21)$$

where $\psi_1 \ldots \alpha_1$ are the functions (13), while $\psi_2 \ldots \alpha_2$ are small 'corrections' of the second order of magnitude.

Introducing (21) in Eqs. (7) to (10) we find that all terms containing $\psi_1 \ldots \alpha_1$, and g_1 vanish on the left side. In the second-order expressions G_i, all terms containing $\psi_2 \ldots \alpha_2$ can be neglected, being of the third order of magnitude. Having regard to (13) we may then write G_i in the form

$$G_i = H_i + M_i \sin 4\mu + N_i \cos 4\mu, \quad . \quad . \quad . \quad (22)$$

where H_i, M_i, N_i are known functions of a, composed of sums of products of the second order of magnitude.

The above procedure is analogous to that used in the theory of 'tides of the second order.'

The method of evaluation of H_1 may be illustrated by the following example:

One of the terms of Eq. (3) may be written $\left(\text{noting that } \frac{\partial}{\partial t} = \Omega \frac{\partial}{\partial b}\right)$:

$$-R(\Psi^2 - \Omega^2)\frac{\partial R}{\partial b} = -(a + z_1)\left[(\Omega + \dot{\psi}_1)^2 - \Omega^2\right]\frac{\partial z_1}{\partial b} = -2\Omega^2 a \frac{\partial \psi_1}{\partial b}\frac{\partial z_1}{\partial b} + T_3,$$

where T_3 is a sum of third-order terms. Treating the other terms in (3) in like manner and collecting all second-order terms thus obtained, it can be shown that

$$H_2 = D = A_\alpha B_p - A_p B_\alpha \quad . \quad . \quad . \quad (23)$$

From Eq. (5) we find by the same procedure

$$H_4 = (\kappa - 1)\Omega D. \quad . \quad . \quad . \quad . \quad (24)$$

The expressions for H_1 and H_3 are more complicated.

Abbreviating

$$M_i \sin 4\mu + N_i \cos 4\mu = \Pi_i$$

we may now write for $\psi_2 \ldots \alpha_2$ the following equations :

$$\ddot{z}_2 - 2\Omega\, a\, \dot{\psi}_2 + \alpha_0 \frac{\partial p_2}{\partial a} - g\, \rho_0\, \alpha_2 + g \frac{\partial z_2}{\partial a} = H_1 + \Pi_1, \quad . \quad . \quad (25)$$

$$a^2 \ddot{\psi}_2 + 2\Omega\, a\, \dot{z}_2 + \alpha_0 \frac{\partial p_2}{\partial b} + g \frac{\partial z_2}{\partial b} = D + \Pi_2, \quad . \quad . \quad (26)$$

$$\frac{z_2}{a} + \frac{\partial z_2}{\partial a} + \frac{\partial \psi_2}{\partial b} - \rho_0\, \alpha_2 = H_3 + \Pi_3, \quad . \quad . \quad (27)$$

$$\epsilon_2 - \kappa\, p_0\, \dot{\alpha}_2 - \alpha_0\, \dot{p}_2 = (\kappa - 1)\, \Omega\, D + \Pi_4 \quad . \quad (28)$$

Any solution must be periodic in b with period 2π and must therefore be included in expressions of the form

$$x_2 = h_x + \sum_{n=1}^{n=\infty} K_{nx} \sin (n\, b + k_{nx}),$$

($x = \psi, z, \alpha, p, \epsilon$), where h_x, K_{nx}, k_{nx} are functions of a and t. This general form comprises a particular solution of the form

$$x_2 = h_x + m_x \sin 4\mu + n_x \cos 4\mu = h_x + \pi_x,$$

where m_x and n_x are functions of a. Introducing this form of x_2 in Eqs. (25 to 28) we obtain three sets of equations, for h_x, m_x and n_x respectively. The equations for m_x and n_x are similar to Eqs. (15) to (18) and may be treated in like manner. The equations for h_x are different. They are found by introducing in Eqs. (25) to (28) h_x for x_2 on the left and deleting Π_i on the right. We may try to determine h_x in the form

$$h_x = (x)_0 + (x)_1\, t + \ldots + (x)_n\, t^n + \ldots, \quad . \quad . \quad (29)$$

where the $(x)_n$ are functions of a. Introducing these expressions into Eqs. (25) to (28) we obtain a system of linear equations for the $(x)_n$ and their derivatives.

It is of interest to note that the power series (29) cannot all be uniformly convergent. This can be seen from Eq. (26). Introducing h_x on the left, deleting Π_2 and integrating we obtain

$$a^2\, \dot{h}_\psi + 2\, \Omega\, a\, h_z = D\, t + F\,(a).$$

Obviously, if the series (29) for h_ψ and h_z were both uniformly convergent, the left-hand side would have an upper bound when $t \to \infty$, whereas on the right side we have $Dt \to \infty$. A further examination of the equations obtained from (25) to (28) by putting $x_2 = h_x$ and deleting Π_i shows that at least two of the quantities h_x must tend to infinity when $t \to \infty$, if $D \neq 0$.

We shall consider the special case where h_ϵ is a given quantity independent of t, $h_\epsilon = f_\epsilon\,(a)$. The heat supplied to a unit mass per unit time is then

$$\frac{\partial q}{\partial t} = \frac{1}{\kappa - 1}\,(\epsilon_1 + f_\epsilon + \pi_\epsilon)$$

and the heat supplied during a period t_0 is $f_\epsilon\, t_0/(\kappa - 1)$. Let us seek a particular solution having the following property : If $D = 0$ and $f_\epsilon = 0$, then all h_x shall be independent of t. It is easily shown that, with these conditions (29) reduces to

$$\begin{aligned} &h_\epsilon = f_\epsilon\,(a), \quad h_z = l_z\, t, \quad h_\psi = l_\psi\, t^2 \\ &h_p = l_p\, t + r_p, \quad h_\alpha = l_\alpha\, t + r_\alpha, \end{aligned} \quad . \quad . \quad (30)$$

where k_ψ, l_x, r_x are functions of a. We find $k_\psi = (\frac{1}{2}\, D - \Omega\, a\, l_z)\, a^{-2}$, and by means of

Eqs. (25) to (28) we can find an equation $F(l_z'', l_z', l_z, D, D', f_\epsilon, f_\epsilon') = 0$, F denoting a linear form. This equation and the boundary conditions determine l_z, D and f_ϵ being given, and l_α, l_p are determined by l_z. The following cases may be noted :

(i) In the non-tilting wave (11) we have $D = 0$. If also $f_\epsilon = 0$, we find $k_\psi = 0$, $l_x = 0$. Thus the wave suffers no secular change, the time t entering only into the periodic terms of the first and second order.

(ii) If, in the case (11) we have $f_\epsilon \neq 0$, the terms in (30) have a simple physical interpretation. The change in pressure and specific volume determined by l_p and l_α, and the vertical velocity l_z are due to the uniform heating (the same for all particles in a horizontal plane) which is superposed on the periodic heating determined by ϵ_1. The horizontal displacement $k_\psi t^2$ is the Coriolis effect of the vertical velocity l_z.

(iii) In the tilting wave (13) we generally have $D \neq 0$. If $f_\epsilon = 0$, the expressions for h_x have the same form (30) as before, but k_ψ, l_x now depend solely on D and no longer have the simple interpretation of the previous case.

(iv) In the case $D \neq 0$, $f_\epsilon \neq 0$ the expressions (30) give the combined effect of the non-periodic heating and the function D, which is determined by the wave (13).

It should be noted that, in the cases (ii), (iii) and (iv), the secular terms in (30) cannot all vanish. It is possible by a special choice of f_ϵ to make, for instance, $l_z = l_\alpha = 0$ but l_p and k_ψ will then be different from zero. In non-linear mechanics it is possible in many cases to avoid secular terms by introducing oscillation periods and wave velocities depending on the amplitude. This method is, however, not applicable in the case studied here, period and wave velocity being constants prescribed in advance.

Since, by hypothesis, the h_x are of the second order of magnitude, the solution (30) can be valid only for a limited time interval.

4. Supply and transport of energy related to the tilt of the wave

The supply of energy is partly of thermal, partly of tidal origin. In order to find an expression for this energy we consider the vector $\mathbf{V} = (p_0 + p_1 + p_2)(\mathbf{v}_1 + \mathbf{v}_2)$ where $\mathbf{v}_1$ and $\mathbf{v}_2$ denote the first-order and the second-order part of the velocity vector. Neglecting third-order terms we write : $\mathbf{V} = p_0 \mathbf{v}_1 + p_1 \mathbf{v}_1 + p_0 \mathbf{v}_2$. We apply the theorem of Gauss to $\mathbf{V}$ between the earth's surface $a = a_0$ and the surface $a = a_1$ (assuming the canal to be of unit width), and then perform a time integration over a whole number of periods. This operation makes the first part $p_0 \mathbf{v}_1$ vanish. Considering the second part $p_1 \mathbf{v}_1$ we may write (dropping the subscripts)

$$\int_{a_0}^{a_1} \int_0^{2\pi} \operatorname{div}(p\,\mathbf{v})\, a\, da\, db = a_1 \int_0^{2\pi} (p\,\dot{z})_{a=a_1}\, db \quad . \quad . \quad (31)$$

Having regard to Eq. (9) we may write

$$\operatorname{div}(p\,\mathbf{v}) = p \operatorname{div} \mathbf{v} + \mathbf{v} \,.\, \nabla p = p\,\rho_0\,\dot{\alpha} + \dot{z}\,\frac{\partial p}{\partial a} + \dot{\psi}\,\frac{\partial p}{\partial b} \quad . \quad . \quad (32)$$

The two last terms are connected with the tidal energy as follows : multiplying Eq. (7) by $\dot{z}$, (8) by $\dot{\psi}$, (9) by $g\dot{z}$, integrating over m periods and combining the three equations thus obtained, we get

$$W(a) = \int_0^{mt_0} (g_1'\,\dot{z} \cos 2\mu - 2 g_1\,\dot{\psi} \sin 2\mu)\, dt = \alpha_0 \int_0^{mt_0} \left(\frac{\partial p}{\partial a}\,\dot{z} + \frac{\partial p}{\partial b}\,\dot{\psi}\right) dt \quad . \quad (33)$$

This is the work performed on a unit mass by the tidal forces. Having regard to (31), (32) we may write :

$$F_1(a_1) = 2\pi a_1 \int_0^{mt_0} (p\,\dot{z})_{a_1}\,dt = \int_{a_0}^{a_1}\int_0^{2\pi} \rho_0 \left(\int_0^{mt_0} p\,\dot{\alpha}\,dt + W(a) \right) a\,da\,db \quad . \quad (34)$$

Treating div $(p_0\,\mathbf{v}_2)$ in like manner we find

$$F_2(a_1) = 2\pi a_1 \int_0^{mt_0} (p_0 \dot{z}_2)_{a_1}\,dt = \int_{a_0}^{a_1}\int_0^{2\pi} \rho_0 \left(p_0\,[\alpha_2] - g\,[z_2] \right) a\,da\,db, \quad . \quad . \quad (35)$$

where $[x_2] = x_2(mt_0) - x_2(0)$.

According to (30) and other possible solutions of Eqs. (25) to (28) the terms in (35) are of the same order of magnitude as the terms in (34), and increase indefinitely when $m \to \infty$. Our equations are thus incompatible with a conservative system. If we may assume that this is due to the simplifying assumptions made (omission of third-order terms and frictional terms) so that, in reality, $\alpha_2(mt_0)$ and $z_2(mt_0)$ have an upper bound when $m \to \infty$, then the terms in (34) become predominant when m increases.

The term on the left in (34) is then the work performed by the surface (14) (with $a = a_1$). This term, given in a different form by Weekes and Wilkes (1947), represents the transport of energy through the level $a = a_1$, maintained by the oscillating motion.

The first term on the right in (34) represents the work performed by the air masses due to the periodic expansion and contraction. Using the expressions (13) for p and α and Eq. (23) we find :

$$\int_0^{mt_0} p\,\dot{\alpha}\,dt = m\,\pi\,D \quad . \quad . \quad . \quad . \quad (36)$$

Integrating (28), and assuming again that α_2 and p_2 have an upper bound, we get when $m \to \infty$, making use of $\Omega\,t_0 = \pi$:

$$\frac{1}{\kappa - 1} \int_0^{mt_0} \epsilon_2\,dt = \frac{1}{\kappa - 1}\,h_\epsilon\,mt_0 \to \pi\,m\,D, \quad . \quad . \quad (37)$$

which is the heat supplied to a unit mass during the time mt_0. Eq. (37) shows that the source of the first energy term on the right in (34) is the non-periodic part of the second-order heat supply. The first-order ' heat function ' ϵ_1, being periodic, renders no contribution.

The quantity D, which enters into the Eqs. (30) to (37) is closely related to the tilt of the wave. It is immediately seen that, in waves of the form (11) or (12), we have $D = 0$. In waves of the form (13), D is generally different from zero. If $D = 0$, we may introduce an auxiliary function $f = f(a)$ such that $B_p = f.A_p$, $B_\alpha = f.A_\alpha$. Introducing these expressions in Eqs. (15) to (17) we can deduce, after some calculation, a condition of the form $B_z'' + L_1 B_z' + L_2 B_z = 0$, where L_1 and L_2 depend on A_z and g_1. However, since B_z is, by definition, independent of g_1, the fulfilment of this condition, although theoretically possible, would be a mere coincidence. We therefore conclude that $D = 0$ is generally incompatible with a tilting wave.

Having regard to the expressions (13) for z and ψ we find from the first integral in (33) the expression :

$$W(a) = \pi\,(-g_1'\,B_z + 2g_1\,B_\psi). \quad . \quad . \quad . \quad (38)$$

Thus W depends not only on the tidal force but also on the heat supply determined by ϵ, since B_z, B_ψ depend on B_ϵ. W vanishes when $\epsilon = 0$, or when ϵ has the form (11), since $B_\psi = B_z = 0$ in these cases. We conclude that a periodic heat supply, asymmetric relative to the sun's meridian, is necessary in order that energy may be put into the model atmosphere by the tidal forces.

Although the two terms on the right in (34) may be conveniently called the ' thermal ' and the ' tidal ' energy term, respectively, it should be noted that they are interdependent. Since $D = 0$ is incompatible with a tilting wave, both terms generally vanish when $D = 0$. Further we note that, in the general case (13), D depends on ϵ_1, and on g_1, since

A_α, A_p depend on both. It follows then from (34) and (36) that the tidal force enters into the ' thermal ' energy term. Tidal and thermal energy are thus closely interconnected and cannot be separated.

Among the significant variables in the wave motion the most easily observable is the barometric variation p_s at a fixed station. This is different from p, which relates to individual air particles. It can be shown that $p = p_s + p_0' z$, so that p_s can be substituted for p in the integral on the left in (34). Writing

$$p_s = C_s \cos(2\mu + \eta_s), \qquad z = C_z \cos(2\mu + \eta_z)$$

we then find the vertical transport :

$$F_1 = 2\pi^2 a_1 m C_s C_z \sin(\eta_s - \eta_z),$$

whence it follows that the transport is upwards if the maximum elevation of the material surface (14) at a station occurs later than the pressure maximum, the delay not exceeding one half-period. Otherwise the transport is downwards.

In the waves (11) to (12) we have $\eta_s = 0$. In (13) we generally have $\eta_s \neq 0$. Thus we can ascertain if the wave is tilted by observing the barometric variation at stations situated at different levels, as mentioned in the introduction.

5. Concluding remarks

The vertical propagation of tidal and thermal energy in a frictionless model atmosphere has been studied by Wilkes. He has shown that, under certain conditions, which seem to be fulfilled in the 12-hr solar oscillation, the energy introduced into the lower atmosphere by tidal forces and thermal action, may be trapped as though by a barrier existing at an upper level. A conclusion of the present paper is that the wave must have a tilt throughout the layer where the vertical transport of energy takes place, while the ' barrier ' must coincide with a layer where the tilt approaches zero. Further it seems that the presence of a thermal excitation, asymmetrical with respect to the sun's meridian, is necessary to establish the energy transport. Once established in this way, the transport comprises both tidal and thermal energy.

The frictionless model and the perturbation method have been widely used in tidal theory; and have proved satisfactory in the treatment of non-tilting waves. In the study of tilting waves, however, the shortcomings of the model and the method become apparent through the occurrence of secular terms like those of Eqs. (30). The horizontal acceleration $2k_\psi$ in (30) would probably be counterbalanced by frictional forces. The question whether the other secular terms are due to the omission of frictional terms in the basic equations or to the limitation of the perturbation method must be left for future examination. Notwithstanding its defects the model is, however, sufficiently realistic to bring out some main points concerning tilt and energy transport.

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Note on energy transfer and conversion in large atmospheric disturbances

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SUMMARY

An analysis is given of the rate of production of perturbation kinetic energy, with emphasis on the influence of static stability, baroclinicity and of heating and cooling.

1. INTRODUCTION

In his contribution to the *Compendium of Meteorology*, R. Fjørtoft (1951) has emphasized the importance of the mean temperature decrease towards the pole, the existence of which is one of the requirements for the conversion of potential energy into kinetic energy of the large-scale eddies. Thus, the baroclinicity of the atmosphere appears to be essential for the development and maintenance of the large disturbances.

Fjørtoft's argument is based on the concept of mixing length associated with a conservative property (namely, entropy). It is doubtful whether the use of this concept is appropriate to the problem under consideration. Moreover, this important result may easily be established within the framework of the perturbation theory.

In order to simplify the mathematical treatment, we shall neglect the curvature of the earth and make use of the classical local Cartesian axes x, y, z, positive respectively to the east, the north and the zenith. The basic current, defined by the velocity field $\mathbf{V}$, the pressure field P and the mass field Q, will be considered as the zonally averaged motion (Fjørtoft's orderly motion) so that the zonal mean values of the corresponding quantities, $\mathbf{v}$, p, q, defining the superimposed perturbation (Fjørtoft's irregular flow), are identically equal to zero. Then, and only then, is the zonal mean value of the kinetic energy $\frac{1}{2}(\mathbf{V}+\mathbf{v})^2$ of the resultant air motion equal to the sum of the kinetic energy $\frac{1}{2}\mathbf{V}^2$ of the zonally averaged motion and the zonal mean value of the kinetic energy $\frac{1}{2}\mathbf{v}^2$ of the large-scale eddies.

Multiplying the Eulerian perturbation equation in vector form scalarly by $\mathbf{v}$, we obtain the equation of kinetic energy $\frac{1}{2}\mathbf{v}^2$ of the large-scale eddying motion

$$Q\left[\frac{\partial \frac{1}{2}\mathbf{v}^2}{\partial t}+\mathbf{V}\,.\,\nabla\tfrac{1}{2}\mathbf{v}^2+\mathbf{v}\,.\,(\mathbf{v}\,.\,\nabla\mathbf{V})\right]=-\mathbf{v}\,.\,\nabla p+q\,\mathbf{v}\,.\,\frac{\nabla P}{Q}. \qquad (1)$$

Substituting in (1) the continuity equation of the basic flow, we obtain the corresponding equation of balance

$$\frac{\partial}{\partial t}(\tfrac{1}{2}Q\mathbf{v}^2)+\operatorname{div}(\tfrac{1}{2}Q\mathbf{v}^2\mathbf{V}+p\,\mathbf{v})=p\operatorname{div}\mathbf{v}+q\,\mathbf{v}\,.\,\frac{\nabla P}{Q}-Q\,\mathbf{v}\,.\,(\mathbf{v}\,.\,\nabla\mathbf{V}), \qquad (1')$$

where the second member expresses the rate of production, within the unit of volume, of kinetic energy of the large-scale disturbances.

Let us examine and interpret in turn the three terms of the second member of the equation of balance (1').

2. Kinetic energy transfer

It has been repeatedly shown that $Q\,\mathbf{v}\,.\,(\mathbf{v}\,.\,\nabla\mathbf{V})$ represents the rate of transfer of kinetic energy from the eddy motion to the mean flow. When integrated over a large volume τ, isolated and axially symmetrical with respect to the earth's axis, the sense of the transfer is determined by the sign of the correlations between the eddy velocity components (v_x, v_y, v_z) and the sign of the horizontal and the vertical wind shears. But, the basic flow being mainly zonal ($V_x \equiv U$, $V_y \cong 0$, $V_z \cong 0$) and the kinetic energy of the mean meridional circulations relatively much smaller, the approximation

$$Q\,\mathbf{v}\,.\,(\mathbf{v}\,.\,\nabla\mathbf{V}) \cong Q\left(v_x v_y \frac{\partial U}{\partial y} + v_x v_z \frac{\partial U}{\partial z}\right)$$

is justified. Moreover, scale considerations (Charney 1948) show that the vertical transfer $Q\,v_x v_z \dfrac{\partial U}{\partial z}$ is at least one order of magnitude less than the horizontal transfer $Q\,v_x v_y \dfrac{\partial U}{\partial y}$. The formula

$$Q\,\mathbf{v}\,.\,(\mathbf{v}\,.\,\nabla\mathbf{V}) \cong Q\,v_x v_y \frac{\partial U}{\partial y} \simeq 10^{-6} \text{ (in MTS units)} \quad . \quad . \quad (2)$$

therefore gives an acceptable approximation for the kinetic energy transfer from the large-scale irregular flow to the zonal mean flow. This transfer, being mainly horizontal, depends in the first place on the horizontal shear of the zonal mean flow U and hence exists even in the absence of baroclinicity. Therefore, in accordance with a recently introduced terminology, this energy transfer is called a ' barotropic process.'

If the earth's curvature is taken into account, the approximate formula (2) assumes the form

$$Q\,\mathbf{v}\,.\,(\mathbf{v}\,.\,\nabla\mathbf{V}) \cong Q\,v_x v_y \left(\frac{\partial U}{\partial y} + \frac{U}{r}\tan\phi\right) \equiv Q\,v_x v_y \cos\phi \frac{\partial}{\partial\phi}\left(\frac{U}{r\cos\phi}\right) \quad . \quad (2')$$

ϕ being the latitude, r the distance from the earth's centre and where $\dfrac{\partial}{\partial y} \equiv \dfrac{1}{r}\dfrac{\partial}{\partial\phi}$.

It is now well established that, in low and middle latitudes, the correlation along the latitude circles between the zonal and meridional components v_x and v_y of the perturbation velocity $\mathbf{v}$ is positive and diminishes rapidly towards the pole in middle latitudes (various papers by J. Bjerknes, V. P. Starr *et al.*, 1948-1952). On the other hand, the horizontal wind shear $\dfrac{\partial U}{\partial y}$ is positive to the south and negative to the north of the *mean* subtropical jet stream, but the latitudinal change $\dfrac{\partial}{\partial\phi}\left(\dfrac{U}{r\cos\phi}\right)$ of the relative angular velocity is two to three times larger, in absolute value, to the south than to the north of the mean jet (Starr 1953, see Fig. 1a). From these circumstances it follows that the conversion of kinetic energy of the large-scale disturbances into the kinetic energy of the zonal mean flow is rather large to the south of the mean jet stream; to the north however the energy transfer takes place in the opposite sense, but is quite small. When integrated over a large volume τ, axially symmetrical with respect to the earth's axis and with a sufficient extent in latitude, the transfer term (2) or (2') contributes to the generation of kinetic energy of the zonal flow (Starr 1953). According to Starr, the rate of generation is given by the expression

$$\iiint_\tau Q\,v_x v_y r^2 \cos^2\phi \frac{\partial}{\partial\phi}\left(\frac{U}{r\cos\phi}\right) d\lambda\, d\phi\, dr > 0,$$

λ being the longitude.

3. Production of kinetic energy

The meaning of the term $q\,\mathbf{v}\,.\,\frac{\nabla P}{Q}$ becomes clear after substituting the equation of the zonally averaged motion and introducing the hypothesis of quasi-homogeneity

$$-\,q\,\frac{\nabla P}{Q} = q\left[\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\,.\,\nabla\mathbf{V} + 2\,\boldsymbol{\omega}\times\mathbf{V}\right] + q\,\nabla\Phi \cong q\,\nabla\Phi,$$

where $\boldsymbol{\omega}$ represents the earth rotation vector and Φ the geopotential. Whence,

$$q\,\mathbf{v}\,.\,\frac{\nabla P}{Q} \cong -\,q\,\mathbf{v}\,.\,\nabla\Phi = -\,g\,q\,v_z \simeq 10^{-6}\ (\text{in MTS units}), \quad . \quad . \quad (3)$$

where g designates the acceleration of gravity.

The term $q\,\mathbf{v}\,.\,\frac{\nabla P}{Q}$ represents mainly the work performed, per unit volume, by the restoring force $-\,q\,\nabla\Phi$ in the gravity field, along the irregular motion. Thus, this term expresses the rate of conversion of potential energy into kinetic energy of the large-scale eddying motion. When integrated over a volume τ, axially symmetrical with respect to the earth's axis, this conversion term gives a positive contribution to the production of kinetic energy of the disturbances, the correlation between the density fluctuation q and the vertical component v_z of the perturbation velocity $\mathbf{v}$ being, as a rule, negative.

Finally, the term $p\,\mathrm{div}\,\mathbf{v}$ represents the influence of expansion ($\mathrm{div}\,\mathbf{v} > 0$) and of compression ($\mathrm{div}\,\mathbf{v} < 0$) of the air on the rate of production of the kinetic energy of the disturbances. This term may be immediately interpreted as the work done by the air in unit volume against the perturbation pressure in the field of the eddying motion, and it therefore gives the rate of conversion of internal energy (heat) into perturbation kinetic energy. When integrated over the volume τ, the conversion term $p\,\mathrm{div}\,\mathbf{v}$ contributes to the production or the destruction of the kinetic energy of ' the irregular flow ' according to the sign of the correlation between the Eulerian perturbation pressure p and the divergence of the perturbation velocity field $\mathbf{v}$.

The rates of conversion $-\,gqv_z$ and $p\,\mathrm{div}\,\mathbf{v}$ are of the same order of magnitude (10^{-6} kJ m^{-3} sec^{-1}); their contributions to the production of perturbation kinetic energy in the volume τ, are not equally important, however, because the correlation between p and $\mathrm{div}\,\mathbf{v}$ is much lower than the correlation between $-\,q$ and v_z (which is indeed, as a rule, very high). Consequently, the production of perturbation kinetic energy results mainly from the conversion of potential energy.

4. Approximate expressions for the rate of production

In the case of large-scale eddies, the continuity equation assumes the simplified form

$$\mathrm{div}\,\mathbf{v} + \frac{v_z}{Q}\frac{\partial Q}{\partial z} \simeq 0,$$

whence,

$$p\,\mathrm{div}\,\mathbf{v} \cong -\,\frac{p}{Q}\frac{\partial Q}{\partial z}v_z \simeq 10^{-6}\ (\text{in MTS units}). \quad . \quad . \quad (4)$$

From (3) and (4), we may conclude that

$$\Sigma \equiv \int_\tau -\left[\frac{p}{Q}\frac{\partial Q}{\partial z} + gq\right] v_z\,d\tau \quad . \quad . \quad . \quad (5)$$

presents a very good approximation to the amount of potential and internal energy nverted into kinetic energy of the ' irregular flow ' in the isolated volume τ. Substituting (5) the relation $\frac{\theta}{\Theta} = -\frac{q}{Q} + \frac{c_v}{c_p}\frac{p}{P}$ and the hydrostatic equation $\frac{\partial P}{\partial z} + gQ \simeq 0$, we have:

$$\Sigma \equiv \int_\tau \left(\frac{p}{\Theta}\frac{\partial \Theta}{\partial z} + gQ\frac{\theta}{\Theta}\right) v_z \, d\tau, \qquad (5')$$

here c_v and c_p are the specific heats of dry air, Θ the potential temperature of the mean w and θ the corresponding perturbation. The main difference between the expressions) and (5') is the difference of at least one order of magnitude between the stratification $\frac{1}{Q}\frac{\partial Q}{\partial z}$ in (5) and the static stability $\frac{1}{\Theta}\frac{\partial \Theta}{\partial z}$ in (5'). Neglecting in (5') the term in p, we otain Fjørtoft's approximate formula

$$\Sigma \cong \int_\tau g\frac{Q}{\Theta}\theta \, v_z \, d\tau > 0. \qquad (5a)$$

It should be noted that (5a) gives a fair approximation for the conversion of potential nd internal energy into kinetic energy of the large-scale eddies. However, the correlations etween $-q$ and v_z and between θ and v_z are much higher than the correlation between and v_z and hence, Fjørtoft's formula (5a) represents *mainly* the conversion of potential nergy into perturbation kinetic energy.

5. Influence of static stability, baroclinicity and of heating and cooling

In order to demonstrate the influence of the static stability $\frac{1}{\Theta}\frac{\partial \Theta}{\partial z}$, the baroclinicity $\frac{1}{\Theta}\frac{\partial \Theta}{\partial y}$ and of heating and cooling on the production

$$\sigma \equiv -\left(\frac{p}{Q}\frac{\partial Q}{\partial z} + gq\right) v_z \qquad (6)$$

er unit volume of kinetic energy of the large-scale eddies, we must introduce the individual agrangian perturbation pressure p_* and density q_*,

$$p_* = p + \mathbf{r} \cdot \nabla P \quad \text{and} \quad q_* = q + \mathbf{r} \cdot \nabla Q,$$

here $\mathbf{r}$ represents the position vector of the perturbed air particle at time t with respect o the non-perturbed position of the same air particle at the same time.

Employing once more the hydrostatic hypothesis $gQ + \frac{\partial P}{\partial z} \cong 0$, we have:

$$\sigma \equiv \frac{v_z}{Q}\left(q_*\frac{\partial P}{\partial z} - p_*\frac{\partial Q}{\partial z}\right) + \left(\nabla\frac{1}{Q} \times \nabla P\right)_x Q\, r_y\, v_z,$$

r

$$\sigma \equiv g\frac{Q}{\Theta}\theta_* \, v_z + \frac{1}{\Theta}\frac{\partial \Theta}{\partial z} p_* \, v_z - \frac{g}{\Theta}\frac{d\Theta}{dy} Q\, r_y\, v_z, \qquad (6')$$

here

$$\frac{\theta_*}{\Theta} = -\frac{q_*}{Q} + \frac{c_v}{c_p}\frac{p_*}{P}$$

nd

$$\frac{d}{dy} \equiv \frac{\partial}{\partial y} - \left(\frac{\partial P}{\partial y}\Big/\frac{\partial P}{\partial z}\right)\frac{\partial}{\partial z}.$$

The zonal mean value of $Q r_y v_z$ represents an exchange coefficient (ton m^{-1} sec^{-1} which is most probably positive.

The contribution of heating ($\theta_* > 0$) and cooling ($\theta_* < 0$), expressed by the firs term of σ, to the production Σ of the perturbation kinetic energy in the volume τ, is no necessarily positive and may in fact be negative. The contribution of the second term proportional to the static stability, is generally negative. The third term *only* contributes effectively to the generation of perturbation kinetic energy and hence the baroclinicity of the basic flow

$$\left(\nabla \frac{1}{Q} \times \nabla P\right)_x = \frac{1}{Q}\left(\frac{\nabla \Theta}{\Theta} \times \nabla P\right)_x = -\frac{g}{\Theta}\frac{d\Theta}{dy} \simeq f\frac{\partial U}{\partial z} \simeq -\frac{g}{\Theta}\frac{\partial \Theta}{\partial y} > 0,$$

where f designates the Coriolis parameter, plays the essential role in the maintenance of the large atmospheric disturbances. Therefore the production of perturbation kinetic energy is called a ' baroclinic process.'

Assuming adiabaticity ($\theta_* \equiv 0$) and neglecting the terms in Eulerian perturbation pressure p, the correlation between p and v_z being generally low, (6) or (6′) reduces to Fjørtoft's approximate formula

$$\sigma \cong -g\, Q\, v_z\, \mathbf{r} \,.\, \frac{\nabla \Theta}{\Theta}. \quad . \quad . \quad . \quad . \quad (6a$$

When the slope of the perturbation displacement $\mathbf{r}$ is smaller than the slope of the isentropic surfaces Θ = constant of the basic flow, perturbation kinetic energy (Fjørtoft 1951) and entropy (Van Mieghem 1952) are produced.

The condition $v_z\, \mathbf{r} \,.\, \nabla \Theta < 0$ is fulfilled for Eady's unstable cyclone waves (Eady 1949) and hence, during their growth, potential energy is converted into kinetic energy of the cyclones and entropy is generated in the atmosphere.

Finally, I wish to mention that the same results may be deduced from the general energy equations (Van Mieghem 1949, 1952).

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The electric charge of raindrops

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Summary

The observations which are described and discussed in this paper were obtained using equipment designed to record rapidly, and without ambiguity, the electric charge and size of individual raindrops. The electric charge is measured by electrostatic induction on a metal cylinder as the drop falls through. The size of the smaller drops is determined from the time of fall between two such cylinders, while the size of the larger drops is measured by the change of capacity of a parallel-plate condenser as the drop passes between the plates. The point-discharge current, electric-field strength and rain current were also measured.

The observations show that on all occasions the range of charge on a particular size of drop is large, often including both signs, and that this range is not appreciably reduced by considering drops recorded during shorter time intervals. However, it is often found that the average charge and the limits of the range of charge show definite relations to the size of the drop. The variation of average charge is such that there is a certain size for which the average charge is zero; for smaller drops the average charge is opposite in sign to that of the electric field, while the larger drops have average charges of the same sign as that of the electric field.

On some occasions the charges on the drops are too small to be recorded. The field is then usually reversed to a small negative value. It is suggested that the drops are falling from electrically neutral or slightly electrified clouds, the reversal of the field resulting from the splashing of drops on the ground.

The capture of ions by raindrops falling in the region of the point-discharge current below an electrified cloud is examined and it is shown that, if reasonable assumptions are made about the initial charges on the drops, the relation between the charge and size of drops arriving at the ground is similar to that found experimentally. Comparison of theory and experiment indicates that the field increased with height owing to the point-discharge space charge.

The range of charge is attributed to the mixing, while falling, of drops leaving the cloud base in regions of different current density. This requires that the regions of high current density are small areas of the cloud base, probably not exceeding 100 m in diameter.

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1. Introduction

The explanation of the electric charge carried by raindrops has long been a majo problem in atmospheric electricity. Many observations have been made both of th charges on individual drops and of the integrated charge (rain-current) but, until tw recent investigations (Simpson 1949, and Hutchinson and Chalmers 1951), no reasonab theory could be formulated to account for the charges. These latest investigations ind cated that the drops had acquired their charges by capture of ions from the point-discharg current below the cloud.

The investigations of charges carried by individual drops show that no simpl relation exists connecting the charge and size of raindrops with the electric field, althoug some such may be expected. It was felt that if any advance was to be made in the presen investigation it could only be achieved by taking a sample of drops more rapidly tha previous workers had done. The results of the investigation have confirmed this expecta tion and have given much new information on the origin of the charge.

I. Apparatus

2. Measurement of raindrops

The criterion in designing the apparatus to measure the charge and size of individual raindrops was that it should measure rapidly and without ambiguity. The charge is measured by electrostatic induction on an open cylinder as the drop passes through (Gunn 1947). The mass of the smaller drops (0·2 to 2·5 mm diameter) is determined from the time of fall between two such cylinders, the mass of the larger drops (1·5 to 6·0 mm diameter) being measured by the change in capacity of a parallel-plate condenser as the drop passes between the plates. The condenser is included in the tuned circuit of an oscillator and the momentary change in capacity produces a change in frequency which is then detected. A schematic diagram of the apparatus is shown in Fig. 1. The double pulse representing charge (upper trace) and the single pulse representing mass (lower trace) are displayed on a double-beam oscillograph with a horizontal time base.

The method of measurement provides observations at a greater rate than was previously possible, and, since the drops are falling freely, complications due to splashing are avoided.

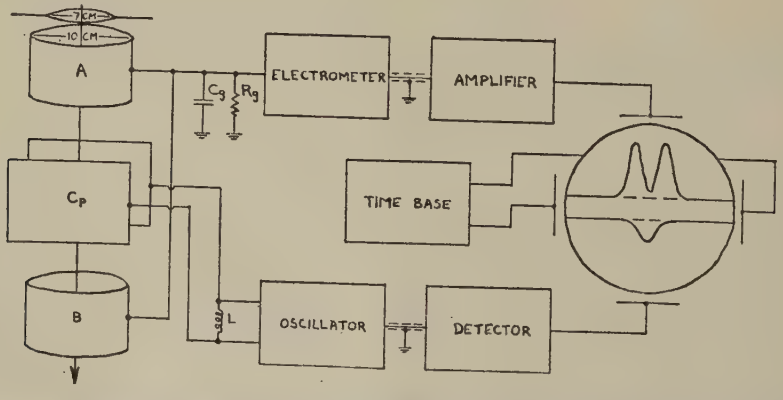


Figure 1. Schematic diagram of the apparatus. The upper trace on the double-beam oscillograph records the electric charge and the lower trace the mass of the raindrops.

(*a*) *Measurement of electric charge*

The two metal cylinders (*A* and *B* in Fig. 1), arranged coaxially, are 5 cm long d 10 cm in diameter; their centres 20 cm apart. The total charge induced on the linders is found (see below) to be 74 per cent of the charge on the drop, over the ntral area (7 cm diameter) through which drops are permitted to fall. An electrometer ve (ME 1400), mounted adjacent to the cylinders, is the first stage of the amplifier. e total input capacity, C_g, is 197 $\mu\mu$f and the input resistance, R_g, is 10^{10} Ω, giving a e constant of 2 sec; this is sufficient for slowly falling drops to be measured. The nal is fed along coaxial cable to a two-stage pentode amplifier in the laboratory. A itch enables the signal to the amplifier to be reduced by a factor of 10 at the input d a potentiometer between the stages gives further control of sensitivity. The gain the amplifier is recorded during each period of measurement by introducing a 50 c/s ibration signal before the potentiometer.

This part of the apparatus is calibrated by determining the total capacity of the linders and the voltage sensitivity of the amplifier. The capacity is found by measuring e reduction in sensitivity when a condenser of known capacity is added to the input cuit, and the voltage sensitivity by applying a known 50 c/s signal to the grid of the ctrometer valve. The fraction of charge induced on a cylinder is found by closing e lower cylinder, in which case the total charge is conducted on to the cylinder.

(*b*) *Measurement of mass*

The change, dC, in the capacity, C, of a parallel-plate condenser, volume V_C, when small sphere, volume V_D and dielectric constant ϵ, is placed inside is given by

$$\frac{dC}{C} = 3\frac{V_D}{V_C}\frac{\epsilon - 1}{\epsilon + 2}, \quad . \quad . \quad . \quad . \quad (2.1)$$

glecting edge effects in the condenser. For water $\epsilon \gg 1$ for all values of conductivity, frequencies up to at least 10^9 c/s. Thus, for raindrops, the change in capacity is pendent only on the volume of the drop, the other quantities being constants of the paratus. The formula has been verified using water drops and lead shot.

The plates of the condenser are 10 cm by 12 cm and the separation is 7 cm. The cillator uses a triode (type 955) and has a frequency of 120 Mc/s. The frequency ange produced by a drop is small (the change is 4·2 kc/s for a drop 6 mm in diameter) ut when the carrier frequency is reduced to 0·5 Mc/s by double superheterodyne (the st I.F. is 10 Mc/s) the relative deviation is increased and can be detected in a phase-scriminator converter. The output of the converter is proportional to the frequency viation and hence to the change in capacity. The apparatus is calibrated directly ing drops of known size.

The dependence of the measured value on the orientation of non-spherical particles as examined using a metal screw 5 mm long and 1·5 mm diameter. The maximum ror in measuring the size was found to be 20 per cent in diameter. As the screw would present a badly distorted drop it is considered that no serious error is introduced by ymmetry of the drops.

The outputs from the two sections of the apparatus are displayed on the double-am oscillograph, the time base, sweep frequency 3 c/s, giving the horizontal scan. The o traces and a time-marker are photographed on 16-mm film moving vertically at nstant speed.

The cylinders and parallel-plate condenser are fitted into a metal box with an erture, diameter 7 cm, in the top. The box is supported on an altazimuth mounting permit measurements to be made on windy days.

3. Auxiliary measurements

Additional apparatus is used to record point-discharge current, and, for the late observations, field strength and rain current. The discharge from an artificial point measured from the potential developed across a resistor (1 MΩ) as the current flow through it. This method, with a larger resistor (10^9 Ω), is also used to measure the curren carried into a collector (aperture 409 cm²) on the raindrops. The field strength is measure by an instrument of the type known as the field-mill, mounted in an inverted positio 2 m above the ground.

The ' effective area ' of the artificial point is estimated to be about 5×10^5 cm². Thi value is obtained by assuming that the relation between natural point-discharge curren density, J, and field strength, X is the same at the observing site in Cambridge as a Kew Observatory where the relation (from Whipple and Scrase 1936, and Simpson 194 is

$$J = 5 \times 10^{-2} (X^2 - X_C^2) \qquad (2.2$$

J being in esu per cm² and X in esu; X_C is the critical field at which discharge com mences. The critical field for the artificial point is 12 v/cm.

The following sign convention will be adopted. Field strength is positive if a positiv charge would experience a downward-directed force. Current is positive if positiv charge is transported downwards or negative charge upwards.

The rainfall is measured continuously by a tilting-siphon raingauge and, on some occasions, by a rate-of-rainfall gauge which records the rate for consecutive periods o one minute.

II. Observations

4. General remarks

The apparatus described in the previous section was set up in an open field near Cambridge, part of the equipment being housed in a small laboratory. The artificial point for the measurement of discharge current is supported, at the end of a vertical rod, 1·4 m above one end of the ridge of the roof. The mounting carrying the drop-measuring apparatus was placed 20 m from the laboratory, near the field-strength meter, the rain-current meter and the raingauge.

Following the installation of the equipment, observations were taken on as many occasions as possible when the rate of rainfall exceeded 1 mm/hr. The 42 periods (between Oct. 1949 and Feb. 1951) include various types of precipitation as shown in Table 1. The total observing time was 313 min in which 9,284 particles were recorded. The greatest rate of measurement was 142 drops in one minute (observation number 30) while, on another occasion, 1,200 drops were measured in 16½ min (number 18).

TABLE 1. Types of precipitation during observation periods.

(a)	Rain showers (duration less than one hour)	
	Non-thundery	1, 3, 5, 7, 8, 9, 13, 20, 22, 26, 31, 32
	Thundery	16, 17, 18, 23, 25, 27, 28, 42
(b)	Continuous rain (duration exceeding one hour)	
	Non-thundery	4, 10, 11, 14, 15, 19, 21, 24, 30, 35, 36, 37, 38, 40, 41
	Thundery	29, 33, 34
(c)	Other than rain	
	Sleet, 2; snow, 6, 39; hail, 12	

When the measurements are examined, it is quickly seen that the method of rapid ording has not succeeded in reducing the range of charge on a particular size of drop. has, however, been possible to analyse the observations on a statistical basis for those asions when a sufficient number of drops was recorded. The minimum number for sfactory analysis is found to be 200 drops. There are 15 periods of observation mbers 1, 7, 8, 14, 15, 18, 21, 23 to 25, 28 to 30, 33 and 34) in which more than this mber were recorded and some of these are described in the following section. Of the er periods, two groups can be distinguished according to the reason why insufficient ps were recorded; first, those occasions on which the drops were not passing straight ough the measuring elements owing to the highly variable wind, and, second, those asions when the charges on the majority of the drops were too small to be detected.

This latter group will be considered briefly. They comprise 19 periods of observation mbers 2 to 5, 9 to 13, 19, 26, 31 and 35 to 41). The value of the smallest detectable arge varied between $0{\cdot}05 \times 10^{-3}$ ESU and $0{\cdot}5 \times 10^{-3}$ ESU and in each case only a small mber of charged drops was detected although it was seen that many drops were passing rough the apparatus. No point-discharge current was detected and these occasions re characterized by negative fields, between -5 and -10 v/cm which show relatively w variation. An example of this type of field variation is shown in Fig. 2. It was corded during continuous rain which persisted from 0800 to 1330 GMT, 19 Feb. 1951; e rainfall-rate was varying between 1 and 3 mm/hr. The short horizontal lines indicate e periods during which drops were measured.

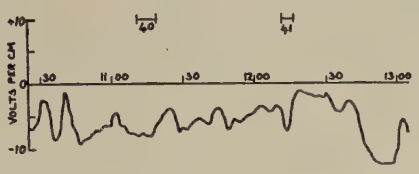


Figure 2. Record of field strength, 8 January 1951.

Some of the 15 periods of observation for which more than 200 drops were measured ill now be described. They are chosen to show the main features of the group. The easurements of charge (in ESU) and size (the diameter in mm, to the nearest 0·05 mm) ave been plotted, each point representing an individual drop. At least one such plot as been made for each period of observation. When many more than 200 drops were easured the time has been divided into consecutive periods each including about 200 rops. Each of these plots of charge and size of the drops will be referred to as a distri- ution. In the examples illustrated the value of the average charge on a particular size f drop (or small range of size) is indicated as a small circle. A small arrow adjacent a point indicates that the trace went off the oscillograph screen at that value; an estimate as used in computing the average charge.

5. Some individual cases

(a) *Number 8, 21 March 1950*

This shower was observed for a period of 24½ min from the beginning. The rate rainfall was 1·5 mm/hr for the first ten minutes; it then increased to 3 mm/hr and ntinued very steadily for the remaining 20 min of the shower. A total of 1,023 drops as recorded. These were divided into five periods, each including about 200 drops. he charge-size distributions for the five periods are shown in Fig. 3.

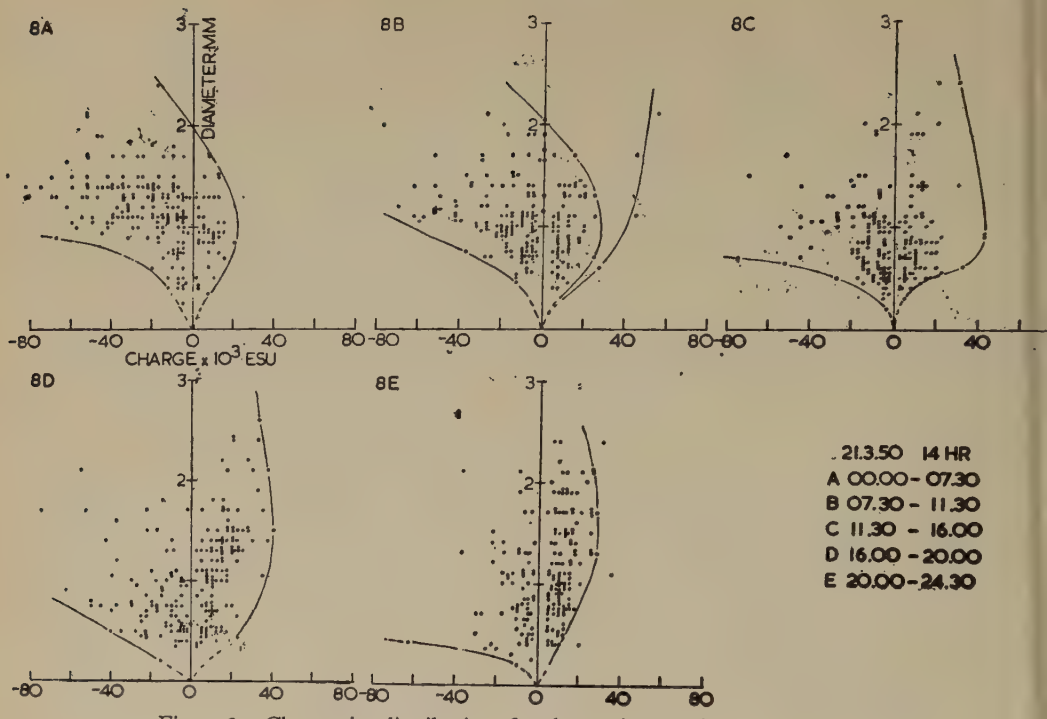


Figure 3. Charge-size distributions for observation number 8, 21 March 1950.

The most striking feature of the distribution is the large range of charges, including both signs (except for the largest drops). However, the limiting charge in all cases shows a well defined relation to size. In distribution 8A, for the first period (7½ min), the positive limit increases to a maximum at 1 mm diameter then decreases, being zero for drops of 2 mm diameter. The limiting negative charge increases with diameter, at first linearly then more rapidly. In distribution 8B, for the second period (4 min) the limits are similar to those of the previous distribution except that four drops fall outside the main part of the distribution. As is indicated in the figure, there appears to be some regular relation between charge and size within the group. In distribution 8C (4½ min) the positive limit shows some change from the previous distributions which is continued in the fourth and fifth distributions (4 and 4½ min). A group of three drops in distribution 8E falls outside the main part.

There is no record of point-discharge current or field strength for this observation.

(b) *Number 18, 21 May 1950*

Observations were taken during a thunder-shower associated with an occlusion. The record of point-discharge current is shown in Fig. 4 (a). Except for short periods coinciding with observed lightning discharges the current was small. The general appearance

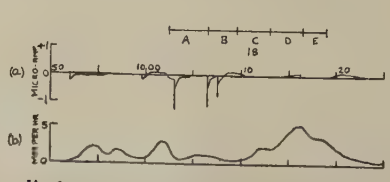


Figure 4. Records of (a) point-discharge current and (b) rate of rainfall for observation number 18.

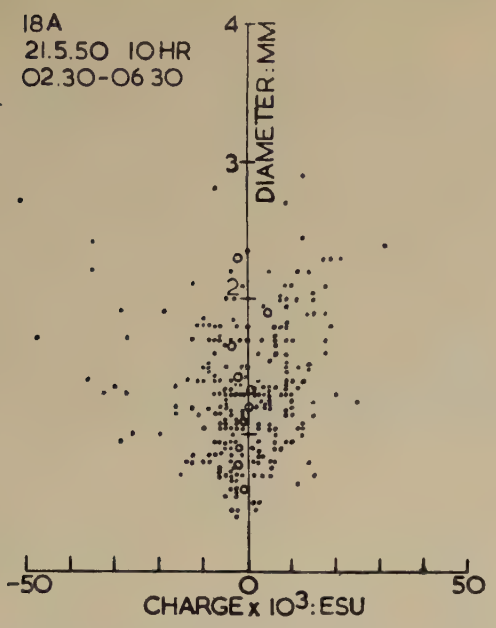


Figure 5. Charge-size distribution A for observation number 18, 21 May 1950.

f the record suggests that the electrical activity was several kilometres distant. The ecord of rate of rainfall, Fig. 4 (b), shows that rain began falling at 0950 BST (0850 GMT) nd increased to a maximum rate of 5 mm/hr. In a period of 16½ min, beginning at 002½ BST, 1201 drops were measured of which 554 were positively charged and 642 egatively charged while 5 had no detectable charge (less than $1{\cdot}2 \times 10^{-3}$ ESU). That uch a large number of drops could be measured was due to the small wind velocity; the xis of the apparatus was vertical. The drops have been divided into five consecutive eriods, of 4, 3, 3½, 3½ and 2½ minutes duration, each including about 240 drops. The harge-size distributions for the first of these periods (18A) is shown in Fig. 5. The istributions for the remaining periods are similar to the one illustrated.

The average charge and limits of charge do not show the regular variation with size hat was noted in the previous case, except for the smallest drops. For these the average harge, which is negative, increases with increasing size as do the limits of charge.

In the distribution illustrated the positive drops carry a total charge of + 3·91 ESU while the negative drops carry a total charge of − 4·52 ESU; the net total charge is thus - 0·61 ESU.

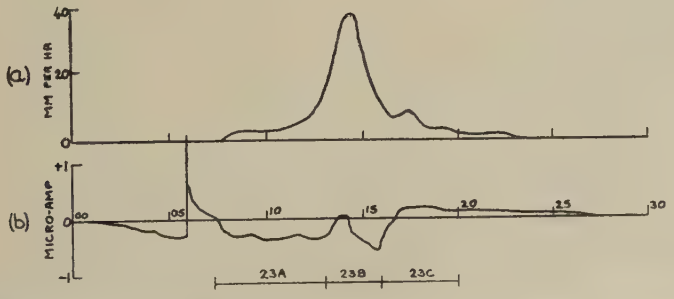


Figure 6. Records of (a) rate of rainfall and (b) point-discharge current for observation number 23.

(c) *Number 23, 21 June 1950*

The rate of rainfall in this shower reached a maximum value of 37 mm/hr, as shown in Fig. 6(a). Rain had begun falling at 1507½ BST and gradually increased to the maximum value at 1514½ BST after which it decreased in intensity, ceasing at 1523½ BST. The record of point-discharge current is shown in Fig. 6(b). Initially the current was mainly negative, being reversed for two short periods, the first associated with a lightning discharge, the second simultaneous with the heaviest rainfall. At 1517 BST the current reversed to a positive value, decreasing to zero at 1527 BST.

The charge-size distributions for drops measured during three periods of the shower are shown in Fig. 7. The gain of the amplifier used in measuring charge was reduced during the second period as so many drops were passing through the apparatus; this has resulted in the measurement of relatively few small drops. The value indicated for the average charge on the small drops in this distribution is therefore unreliable.

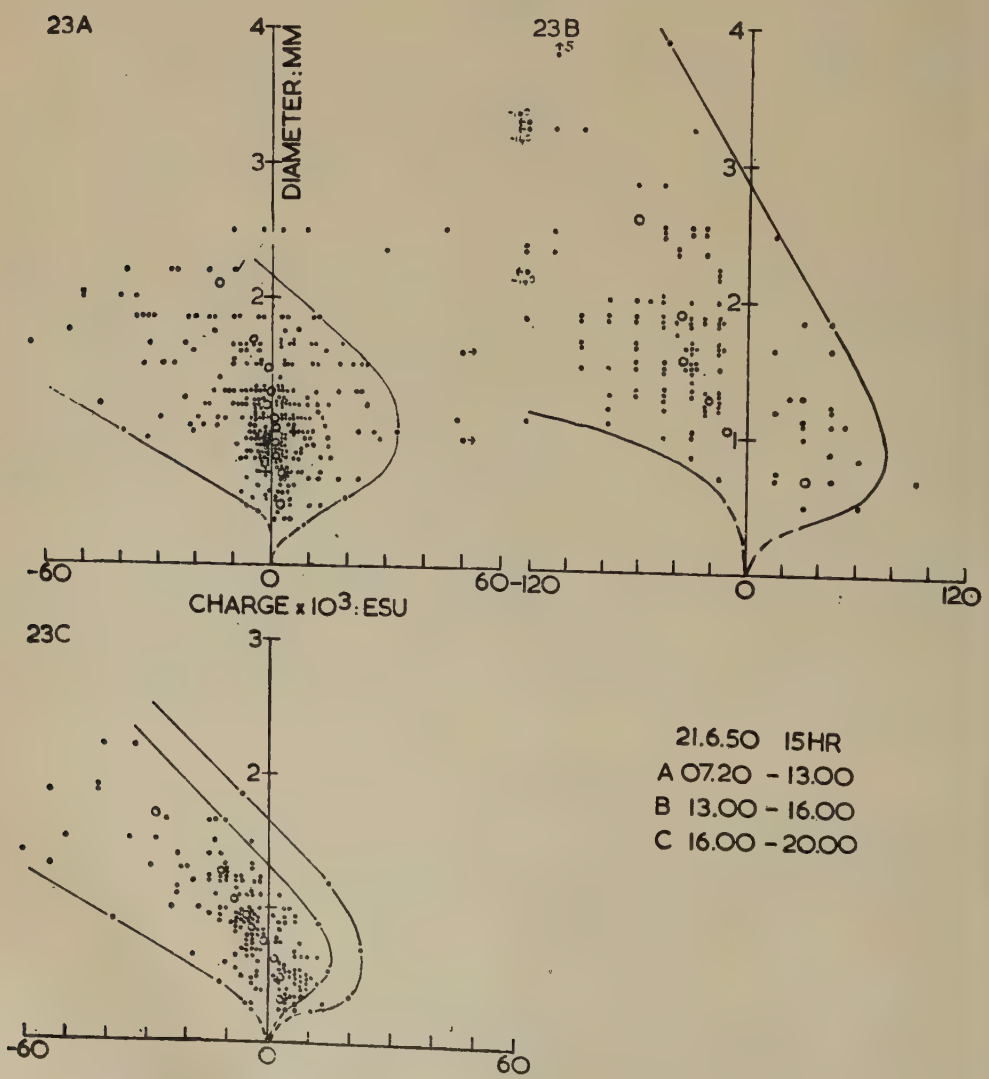


Figure 7. Charge-size distributions for observation number 23, 21 June 1950.

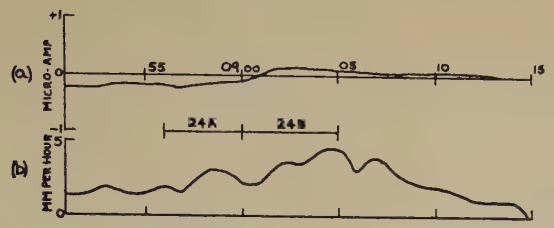


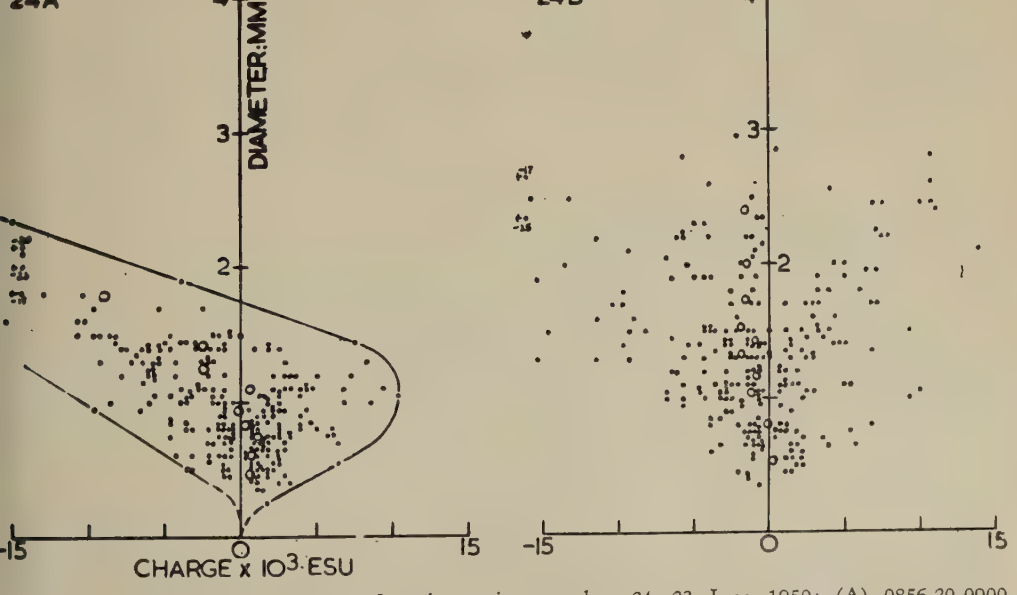
Figure 8. Records of (a) point-discharge current and (b) rate of rainfall for observation number 24.

The three distributions show the same general features as previous distributions; ne average charges on the smaller drops being of one sign and on the larger drops of ne other sign. The third distribution shows a group of six drops outside the main art with a relation of charge and size similar to that of the adjacent limit of charge.

(d) *Number 24, 23 June 1950*

Rain began falling at 0550 BST and continued until 0940 BST. Point-discharge urrent was first recorded at 0823 BST, a small positive current. After five minutes it ecame negative but at 0835½ BST it again was positive. At 0841 BST it became negative nd remained so until 0901 BST. The latter part of the record is shown in Fig. 8 together ith the corresponding record of rate of rainfall.

Observations were made during a period of 9 min from 0856 BST, 490 drops being neasured. The charge-size distributions for the two periods into which the drops were ivided are shown in Fig. 9. The two distributions are quite different in character. The istribution for the first period (number 24A) shows a well defined relation with size or the average charge and the limits of the range of charge. The succeeding distribution number 24B), however, does not show these relations, resembling the distributions of bservation number 18 in this respect. It is noteworthy that the discharge current hanged sign at the beginning of this distribution.



igure 9. Charge-size distributions for observation number 24, 23 June 1950; (A) 0856.20-0900, (B) 0900-0905.

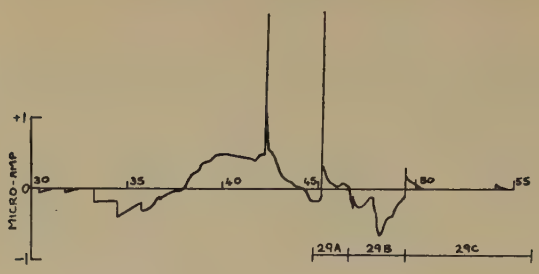


Figure 10. Record of point-discharge current for observation number 29.

The total charge on the positively charged drops is 0·55 ESU for the first period and 0·34 ESU for the second. The corresponding totals for the negatively charged drops are — 0·43 ESU and — 0·60 ESU; the total charges are thus 0·12 ESU and — 0·16 ESU, respectively.

(e) *Number 29, 28 August 1950*

Observations were made during a thunder-shower which preceded a thunderstorm. Rain began falling at 1440 BST and continued at a rate of about 20 mm/hr for 16 min. In a period of 12 min after 1444½ BST 631 drops were measured. The record of point-discharge current, Fig. 10, suggests that the cloud was of negative polarity, if the sudden changes due to lightning discharges are ignored. A curious ' kick ' can be seen on the record at 1447 BST. This was simultaneous with a close lightning discharge; the close discharges at 1442 BST and 1445 BST gave large field changes.

The charge-size distributions for the three periods into which the observations were divided are shown in Fig. 11. This series of distributions shows features which have been noted in the previous examples and one which is peculiar to this observation, namely change of ' sign ' of the distribution. As for many of the other distributions the average charges on the smaller drops are of one sign and the average charges on the larger drops are of the other sign. However, the charges on the smaller drops are, on the average, negative during the first two periods whereas they are, on the average, positive during the third period. The range of charges in the third period shows a reduction from the preceding periods.

6. The range of charge

It is immediately seen that there is no simple relation between the charge and size of the drops. Even on those occasions when the sample of drops was taken during a short period in which the electric field and point-discharge current were steady, or changing slowly, the range of charge on a particular size of drop is large, often including both signs. This is clearly shown in Fig. 12 where the charges on drops within a small range of sizes are plotted against time. The time is taken in half-minute intervals for convenience. The large range of charge is characteristic of all periods of observation, including those in which few drops were measured.

In some cases there is little change in the range of charges during the observation (numbers 8, 18 and 30) but others show a regular change. Thus there may be an increase, as during the initial part of observation number 23 (Fig. 12 (b)), or a decrease, as in observation number 29 (Fig. 12 (d)). In observation number 24 (Fig. 12 (c)) there is evidence of a discontinuous change from an increase to a decrease in the range of charges.

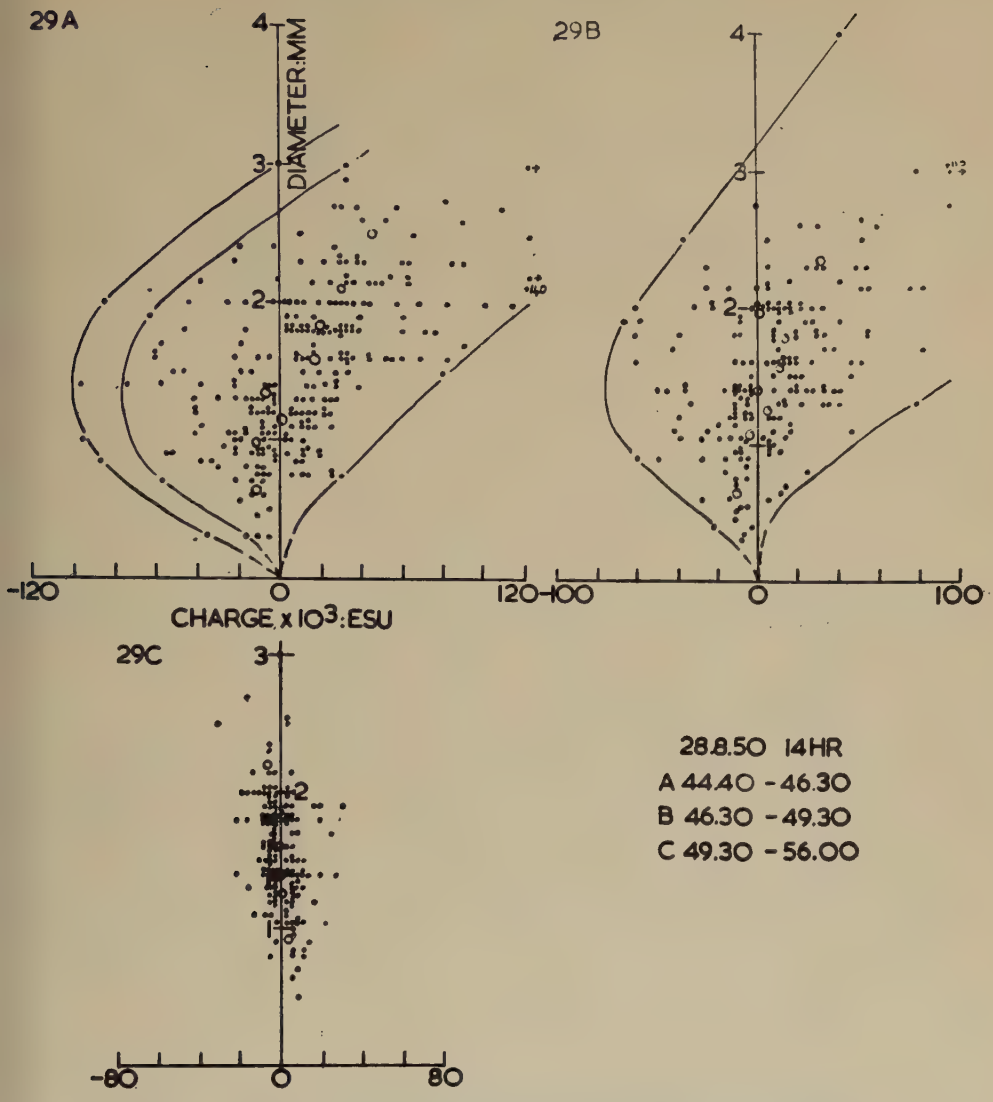
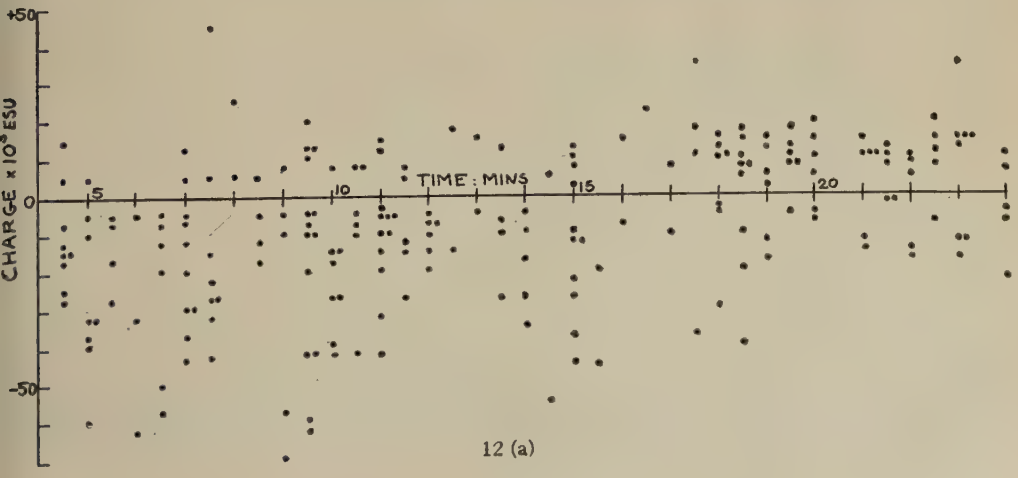


Figure 11. Charge-size distributions for observation number 29, 28 August 1950.



12 (a)

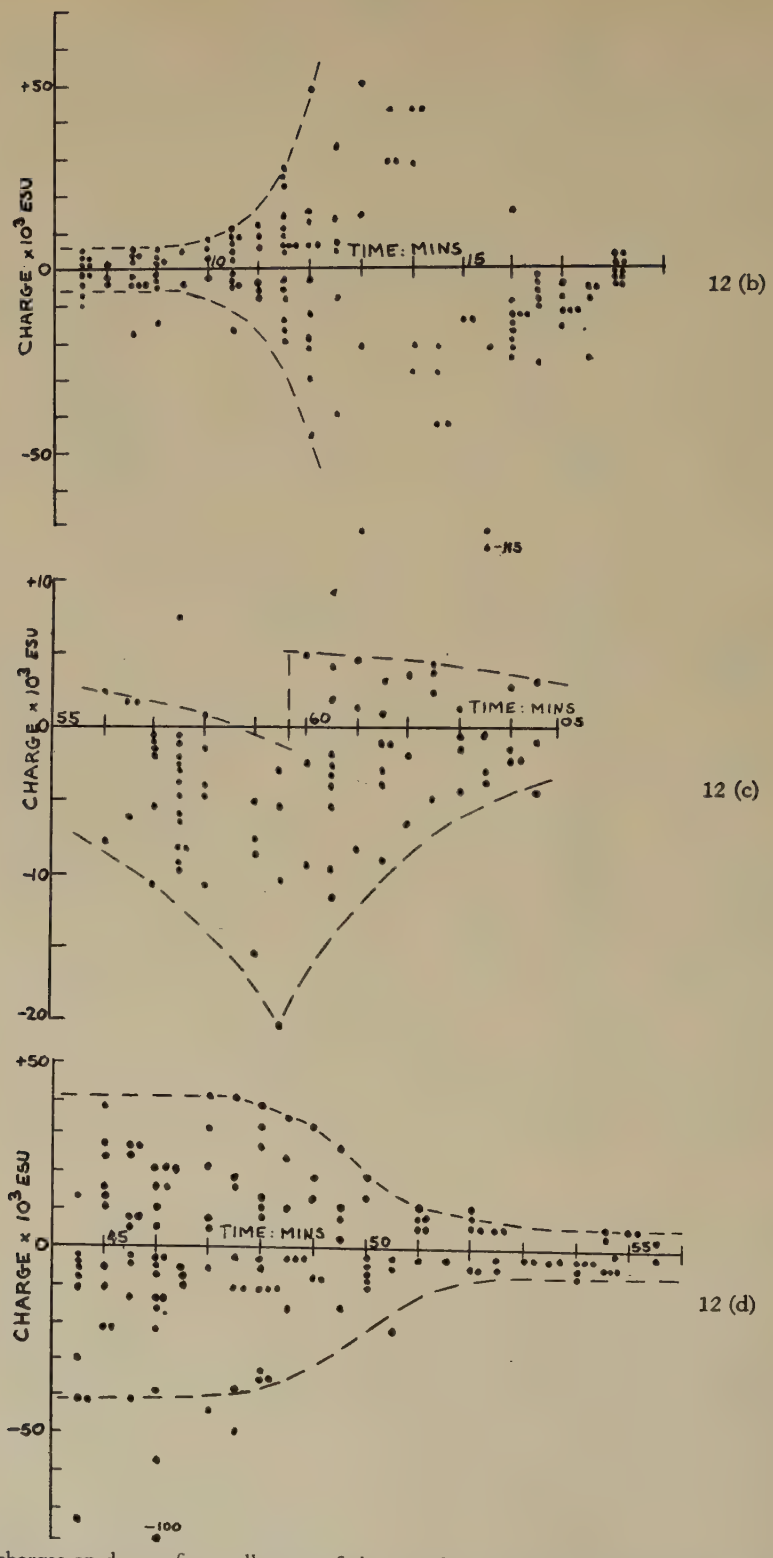


Figure 12. The charges on drops of a small range of size as a function of time; (a) observation number 8, drops 1·0 to 1·2 mm diameter; (b) observation number 23, drops 1·0 to 1·2 mm diameter; (c) observation number 24, drops 1·4 to 1·6 mm diameter; (d) observation number 29, drops 1·2 to 1·4 mm diameter.

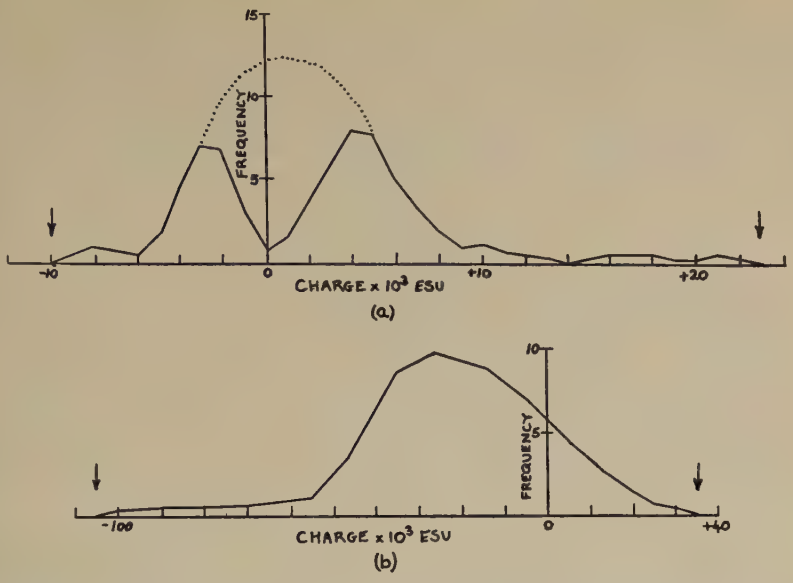


Figure 13. Relative frequency of charges, from distribution number 30A; (a) drops 0·2 to 0·6 mm diameter, showing, dotted, the estimated correction for undetected drops; (b) drops 1·0 to 1·2 mm diameter.

The drops are not uniformly distributed through the range of charge but are concentrated about a particular value of charge. This is shown in Fig. 13 which is derived from distribution number 30A. The charge-frequency graphs are given for two size-ranges (a) drops 0·2 to 0·6 mm in diameter, and (b) drops 1·0 to 1·2 mm in diameter. The graphs for these and other cases are, in general, not symmetrical. Comparison of the two graphs of Fig. 13 illustrates the failure of the apparatus to detect small drops carrying little or no charge. This produces the dip in frequency about the zero of charge and it is suggested that the true frequency is represented by the dotted line.

7. The charge-size relation

When the distributions are examined carefully it is noticed that what is, at first sight, a random relation does show certain regular features concerning the average charge and limits of charge. The general features of a distribution are as shown in Fig. 14. There is a certain size of drop, radius a^*, for which the average charge is zero. Drops smaller than this have average charges of one sign while drops larger than this have average charges of the opposite sign. We shall designate a distribution positive or negative, according to the sign of the charges on the smaller drops. A positive distribution is illustrated; a negative distribution is the mirror-image of this. The average charge, Q^*, on drops of radius $\frac{1}{2}a^*$ should be noted, it will be referred to later.

The limit of charge which shows a similar variation with size to that of the average charge will be called the upper limit of charge and the quantities Q' and a' (see Fig. 14) are defined by analogy with Q^* and a^*. The other limit of charge, called the lower limit, increases with size, in some cases linearly, in others more rapidly, up to a certain size after which it becomes less well defined and decreases. The values of Q^*, a^*, Q' and a' for those distributions for which sufficient drops were measured are collected together in Table 2, with the value of the field at the ground, X_g, and the rate of rainfall, R, both averaged over the period of the distribution.

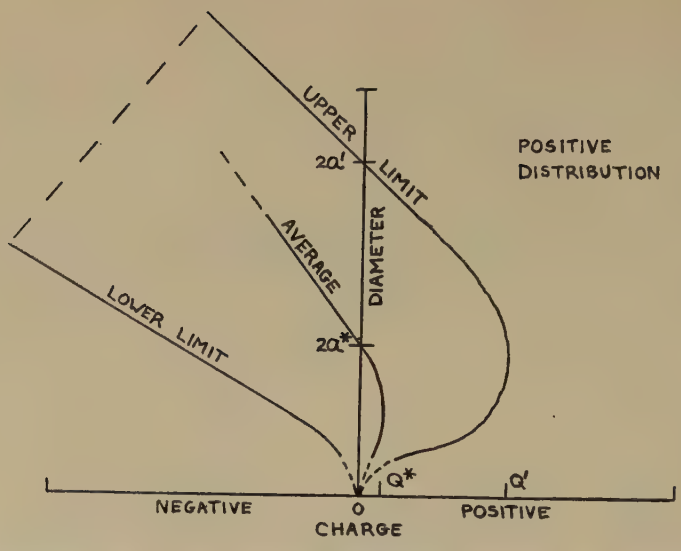


Figure 14. The general form of the charge-size distribution illustrating the nomenclature. The points representing the individual drops fall between the upper and lower limits of charge.

TABLE 2. COLLECTED DATA FOR CHARGE-SIZE RELATION.

Distribution number	$Q^* \times 10^3$ ESU	a^* cm	$Q' \times 10^3$ ESU	a' cm	X_g ESU	R mm/hr
1	− 1·0	0·05	− 20	0·165	0·20	10
6	—	—	22	0·057	− 0·21	5
7A	2·5	0·035	14	0·05	− 0·24	3
7B	0·5	0·025	13	0·065	− 0·13	3
8A	—	—	20	0·09	—	3
8B	—	—	27	0·10	—	3
14B	—	—	125	0·19	—	16
15	—	—	150	0·16	—	16
22	− 1·5	0·06	− 8	0·10	0·17	6
23A	2·0	0·06	35	0·105	− 0·22	4
23B	—	0·05	80	0·14	− 0·26	25
23C	2·0	0·035	22	0·08	0·14	5
24A	1·0	0·05	10	0·08	− 0·15	3
25	− 1·0	0·06	—	—	0·24	30
28	− 3·0	—	− 55	0·15	0·17	5
29A	− 12·0	0·07	− 95	0·14	0·10	20
29B	− 10·0	0·065	− 65	0·14	− 0·19	10
29C	2·0	0·075	25	0·13	0·10	5
30A	2·0	0·04	15	0·07	− 0·15	3
30C	2·0	0·04	27	0·105	0·07	3
30D	—	—	40	0·13	0·10	3
30E	—	—	27	0·08	0·07	3
33	2·0	0·05	15	0·12	− 0·12	3

When the sign of Q^* and Q' for each distribution is compared with the sign of X_g it is seen that there is a tendency for them to be opposite. The exceptions occur during the latter parts of observation numbers 23, 29, and 30. Since the sign of Q^* and Q' was used to define the sign of the distribution we can say that, in general, the distribution is opposite in sign to the field at the ground. Now this field will, for clouds nearly overhead, be determined mainly by the charge in the lower part of the cloud. Thus we find that the larger drops arrive at the ground with charges, on the average, the same

n as the charges in the lower part of the cloud while the smaller drops arrive with erage charges opposite to this. We shall return to the interpretation of this later.

Some distributions cannot be given a definite sign; examples occurred in observation mbers 18, 21 and 24. The average charge on all sizes of drops is small and does not ow the regular variation with size found in other cases. There is nothing to indicate hy these distributions do not show the same general features as the other distributions.

8. Rain current

Two corrections are necessary before the measurements can be integrated to give e rain current. First, owing to the limited acceptance-angle of the apparatus, the spersion of drops by the wind will favour drops of a particular size and distort the op-size distribution. Secondly, the apparatus fails to record drops smaller than 1·5 mm ameter carrying small charges. This latter effect was illustrated in Fig. 13. Though ese two factors do not seriously affect the charge-size relation, which is the main part the present investigation, they may have a profound effect on the apparent value of e rain current. It is found on many occasions that the charge carried by the positive ops is nearly equal to the charge carried by the negative drops, the total charge being nall compared with either and thus sensitive to an error in the separate parts.

The uncorrected observations indicate that the rain current is more frequently of e same sign as the field than of opposite sign, contrary to Simpson's (1949) observation the 'mirror-image' effect. Since the difference could be explained by the failure the present method to record the smallest drops it was decided to measure rain-current irectly. Preliminary observations on three occasions do show the occurrence of the irror-image effect. It is therefore concluded that the charges on the undetected drops ntribute significantly to the rain current.

9. Summary

The main features of the present series of observations of the electric charge of aindrops will now be summarized.

(a) On occasions when the charges on the majority of the drops are too small to e detected the field is usually reversed to a negative value, between − 5 and − 10 v/cm.

(b) The range of charge on any particular size of drop is large, often including oth signs, and is not appreciably reduced by considering smaller time intervals.

(c) When sufficient drops are measured during an observation period to enable statistical analysis to be made it is generally found that the average charge and limits f charge show a regular variation with size. The variation of average charge is such hat there is a certain size for which the average charge is zero; for smaller drops the verage charges are of one sign, opposite to that of the field, while larger drops have average harges of the other sign, the same sign as that of the field.

The remainder of the paper will be devoted to an attempt to explain these observations.

III. Discussion

10. Preliminary remarks

A complete explanation of these and other observations of rain electricity must take into account three stages in the life history of the drops. The first stage, within the cloud, is the growth of the drop and the possible acquisition of electric charge. The second stage is the period of free fall below the cloud during which time the size of the

drop is sensibly unchanged but the charge may be modified by capture of ions. Finally, the third stage is the arrival at the ground with the possibility of electrical effects associated with the violent fracture of the drops. We shall consider the three stages in the reverse order.

The following symbols and units are used :

R = rate of rainfall, mm/hr
j = current density, ESU/cm^2
J = point-discharge current density, ESU/cm^2
X = field strength, ESU
a = drop radius, cm
v = terminal velocity, cm/sec
Q = drop charge, ESU
x = distance upwards, cm
λ = conductivity, ESU
w = ionic mobility, ESU
n, V, k, y and z are defined in the text

The sign convention is as stated in Section 3.

11. Splashing at the ground

The amount of charge generated by splashing water is a variable quantity but is generally about 0·1 ESU/cm^3 (Nolan and Enright 1922). The sign is such that the fragments carry positive charge into the ground while the air is given a negative charge. The rate at which the field at the ground is reduced by this space charge is easily seen to be $-0{\cdot}6R$ v/cm per min. Thus, a few minutes after rain begins falling, the field is reversed from the normal positive value. The negative field can continue to increase, the negative ions of the space charge being carried upwards by turbulent motions against the action of the field, until the critical field for point discharge is exceeded. Equilibrium is established when the rate at which negative charge is supplied by splashing is equal to the rate at which positive charge is supplied by point discharge. The current supplied by splashing is

$$j = R/3{\cdot}6 \times 10^4 \text{ ESU/cm}^2 \quad . \quad . \quad . \quad (11.1)$$

while the point-discharge current is

$$J = 5 \times 10^{-2}\,(X^2 - X_C^2) \text{ ESU/cm}^2 \quad . \quad . \quad (2.2)$$

Equating these currents and expressing the field in v/cm, we find

$$(X^2 - X_C^2) = 5R, \quad . \quad . \quad . \quad . \quad (11.2)$$

which gives the value of the field at equilibrium in terms of the critical field and the rate of rainfall. The equation is shown in graphical form in Fig. 15 for five values of the critical field. For larger values of the critical field the equilibrium field is insensitive to changes in the rate of rainfall. The present series of observations indicate that for those periods when the charges on the drops were undetectable owing to their smallness, and for which the cloud is presumably not sufficiently electrified to cause appreciable disturbance of the field, the field was generally about -7 v/cm. This leads to a value of about -5 v/cm for the critical field, a reasonable value.

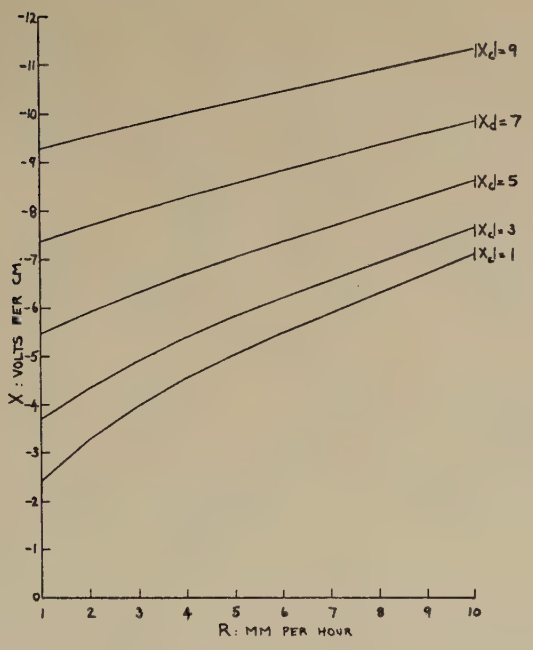


Figure 15. The field strength at equilibrium as a function of the critical field and the rate of rainfall.

Evidence for the existence of the negative space charge close to the ground is provided by observations of the electric field at the Eiffel Tower (height 300 m). It has been found during periods of steady rainfall that the field at the top of the tower is reduced in value, remaining positive in sign, while the field at the ground is reversed to a small negative value (Chauveau 1922).

Instruments which record rain current continuously are subject to this splashing effect. For Cambridge, where the annual rainfall is about 20 in., the charge due to splashing amounts to 17 coulomb/km^2 per yr. (This is close to Wormell's (1930) estimate of 20 C/km^2 per yr as the contribution of precipitation currents to the ' electrical balance sheet '). The compensating point-discharge current is $-$ 17 C/km^2 per yr, which is to be added to the term for point-discharge current in the electrical balance sheet.

12. The capture of ions by raindrops

In the present investigation the measurements of the electric charges of the drops are not affected by splashing although the reversal of the field is recorded. On those occasions when the majority of drops carried charges large enough to be recorded, the field was sufficiently great to produce a discharge current at the artificial point. The effect on the field of the splashing then becomes negligible.

We shall now examine in what way the charges on the drops are modified by the capture of ions from the unipolar discharge current below electrified clouds. The process of capture under the influence of an electric field proposed by Wilson (1929) has been demonstrated by Gott (1933, 1935) and has been treated theoretically by Whipple and Chalmers (1944). The application to the region below clouds is limited by uncertainty in the relevant parameters; we shall therefore consider a simple model and later discuss the deviations which may be encountered in actual cases.

The model will be associated with a bipolar cloud of negative polarity. The lower part of the cloud is, by definition, a region of positive charge and from it positively charged drops are falling. In the region below the cloud the field is positive and there is a positive point-discharge current consisting of an upward flow of negative ions. From Whipple and Chalmers (1944) the rate of charging of a drop due to the capture of ions is

$$3Xa^2 < Q \qquad \frac{dQ}{dt} = -4\pi\lambda Q, \quad . \quad . \quad . \quad . \quad (12.1)$$

$$3Xa^2 > Q > -3Xa^2 \qquad \frac{dQ}{dt} = -\frac{\pi\lambda(3Xa^2 + Q)^2}{3Xa^2}, \quad . \quad . \quad (12.2)$$

$$Q < -3Xa^2 \qquad \frac{dQ}{dt} = 0. \quad . \quad . \quad . \quad . \quad . \quad (12.3)$$

Consider Eq. (12.2). The conductivity at a height x is J/X; thus the equation of charging for the falling drop may be written

$$\frac{dQ}{dx} = \frac{\pi J(3Xa^2 + Q)^2}{3X^2 a^2 v_a}, \quad . \quad . \quad . \quad (12.4)$$

since v_a is the terminal velocity of the drop.

The variation of X and J with height are not, in general, known. We shall here treat two particular cases. In the first we neglect the space charge of the point-discharge current and take X constant with height. The space charge was shown by Wilson (1925) to lead to the following relation

$$X^2 - X_g^2 = 8\pi Jx/w \quad . \quad . \quad . \quad (12.5)$$

This is the second case we shall consider. In both cases J is assumed independent of height.

(*a*) $X = X_0$, a constant. On integrating (12.4) we find that the charge on the drop at the ground is given by

$$\frac{3X_0 a^2}{3X_0 a^2 + Q} = \frac{\pi J x_0}{X_0 v_a} + \frac{3X_0 a^2}{3X_0 a^2 + Q_0} \quad . \quad . \quad (12.6)$$

where Q_0 is the charge on the drop as it leaves the cloud, base height x_0. The initial charges on the drops are not known, so we assume the simple relation

$$Q_0 = 3nX_0 a^2 \quad . \quad . \quad . \quad . \quad (12.7)$$

Substituting this in (12.6) and introducing the parameter

$$V = \pi J x_0 / X_0, \quad . \quad . \quad . \quad . \quad (12.8)$$

we have

$$\frac{3X_0 a^2}{3X_0 a^2 + Q} = \frac{V}{v_a} + \frac{1}{1+n} \quad . \quad . \quad . \quad (12.9)$$

This equation shows that the charge-size relation is determined by two parameters, n representing the initial charge on the drops and V representing the electrical conditions of the region in which the drop is falling. The form of the relation is shown in Fig. 16 for several values of V and in Fig. 17 for several values of n (the analysis is not exact for values of n greater than unity).

The relation between charge and size is, for some pairs of values of V and n, similar to that found experimentally for the average charge and the upper limit of charge. There is a certain size of drop, radius a^*, having zero charge; drops smaller than this carry charges opposite in sign to that of the field while the larger drops have charges of the

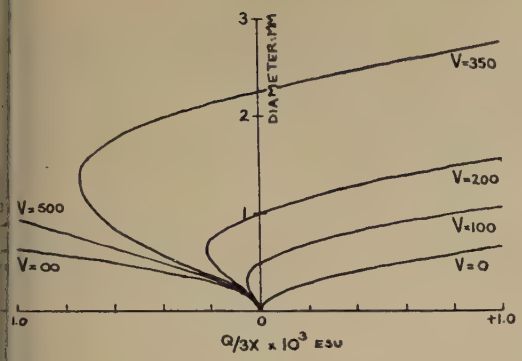


Figure 16. Theoretical charge-size relation at the ground, showing the dependence on V, with $n = 1$.

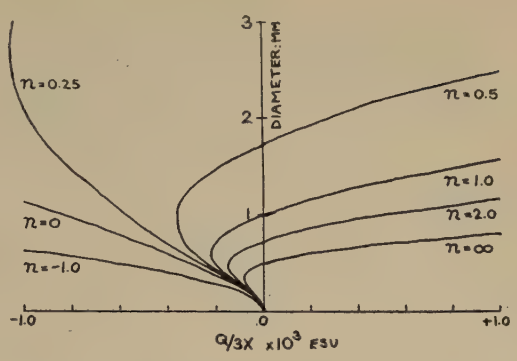


Figure 17. Theoretical charge-size relation at the ground, showing the dependence on n, with $V=200$.

same sign as the field. From sets of curves such as those illustrated it is found that $Q^*/3X_0\,a^*$ is independent of n and proportional to V. Therefore Q^*/a^* is proportional to Jx_0 and independent of n and X_0. We may expect, however, that n, X_0 and J are interdependent through the common agency of the cloud.

This interesting result can be derived analytically as follows. Since a^* is, by definition, the radius of the drop for which the final charge is zero, we have, from (12.9),

$$1 = \frac{V}{v_a} + \frac{1}{1+n} \quad . \quad . \quad . \quad (12.10)$$

There is no simple expression for v_a as a function of a for all sizes of drop; however, over the limited range of values of a between 0·01 and 0·09 cm the terminal velocity and drop radius are related by

$$v_a = ka, \quad . \quad . \quad . \quad . \quad . \quad (12.11)$$

where k has the value of 7,500 sec^{-1}; the maximum error over the stated range is 10 per cent.

Hence, using this relation and assuming $n = 1$,

$$a^* = 2V/k \quad . \quad . \quad . \quad . \quad (12.12)$$

The radius of the drop whose final charge is Q^* is, by definition, $\frac{1}{2}a^* = V/k$; thus from (12.9) we have

$$Q^* = -X_0\,a^{*2}/4 \quad . \quad . \quad . \quad (12.13)$$

Therefore

$$Q^*/a^* = -\pi Jx_0/2k \quad . \quad . \quad . \quad (12.14)$$

showing the dependence of Q^*/a^* on Jx_0 and independence of X_0.

Substituting the value for k we obtain

$$Q^*/a^* = -2{\cdot}0 \times 10^{-4}\,Jx_0 \quad . \quad . \quad . \quad (12.15)$$

or

$$Jx_0 = -5{\cdot}0 \times 10^3\,Q^*/a^* \quad . \quad . \quad . \quad (12.16)$$

If n is retained in the calculation we find

$$\frac{Q^*}{a^*} = -\frac{\pi Jx_0}{2k}\left\{\frac{3\ (n+1)}{2\,(2n+1)}\right\} \quad . \quad . \quad . \quad (12.17)$$

The factor in brackets varies between 1·5 ($n = 0$) and 0·75 ($n = \infty$) showing the insensitivity of (12.14) to variation in n.

(b) $X^2 - X_g^2 = 8\pi Jx/w$. In this case, in addition to including the effect of the space charge of point-discharge ions we shall also use the exact charging equations.

Substituting for X in (12.4) and neglecting X_g we obtain

$$\frac{dQ}{dx} = \frac{3a^2\pi J}{v_a}\left\{1 + \frac{Q}{3a^2}\left(\frac{w}{8\pi J}\right)^{\frac{1}{2}}\frac{1}{x^{\frac{1}{2}}}\right\}^2 \quad . \quad . \quad (12.18)$$

By putting

$$y = \frac{wQ}{12a^2 v_a} \quad . \quad . \quad . \quad . \quad (12.19)$$

and

$$z = \frac{(\pi Jxw)^{\frac{1}{2}}}{2^{\frac{1}{2}} v_a} \quad . \quad . \quad . \quad . \quad (12.20)$$

the charging equation may be reduced to the form

$$z > y > -z \qquad \frac{dy}{dz} = \frac{(y+z)^2}{z} \quad . \quad . \quad . \quad . \quad (12.21)$$

This equation cannot be integrated in simple terms but is suitable for evaluation by numerical methods.

Eq. (12.1) becomes, in reduced form

$$y > z \qquad \frac{dy}{dz} = 4y \quad . \quad . \quad . \quad . \quad . \quad (12.22)$$

which may be integrated directly, while (12.3) becomes

$$y < -z \qquad \frac{dy}{dz} = 0 \quad . \quad . \quad . \quad . \quad . \quad (12.23)$$

By analogy with the previous case we assume as the initial charge-size relation

$$Q_0 = 3nX_0a^2 \quad . \quad . \quad . \quad . \quad (12.24)$$

where $X_0 = (8\pi Jx_0/w)^{\frac{1}{2}}$ is the field strength at the cloud base. In reduced form the initial values are given by

$$y_0 = nz_0 \quad . \quad . \quad . \quad . \quad . \quad (12.25)$$

The charge-size distribution at the ground may now be evaluated for pairs of values of n and Jx_0. The distributions are found to be similar to those of the previous section which were illustrated in Figs. 16 and 17. It is also found that Q^*/a^* is proportional to Jx_0 and independent of n, a result which we shall now show analytically following the method of the previous section.

In Fig. 18 curve A is the solution of (12.21) and (12.22) passing through the point $y = z = 0$. The intersection of this curve with the line $y = nz$ gives the initial values of y and z representing a drop (radius a^*) which arrives at the ground with its charge reduced to zero. For $n = 1$ the values are found to be $y_0 = z_0 = 0{\cdot}86$. Using, as before, $v_a = ka$, it is seen that the initial values for a drop of radius $\frac{1}{2}a^*$ are $2y_0$ and $2z_0$; that is, for $n = 1$, $y_1 = z_1 = 1{\cdot}72$. Curve B is the solution of (12.21), (12.22) and (12.23) passing through the point (y_1, z_1). The intersection of this curve with the line $z = 0$ gives the value of y representing the charge Q^* on this size of drop at the ground. For $n = 1$ it is found that $y_2 = -0{\cdot}32$.

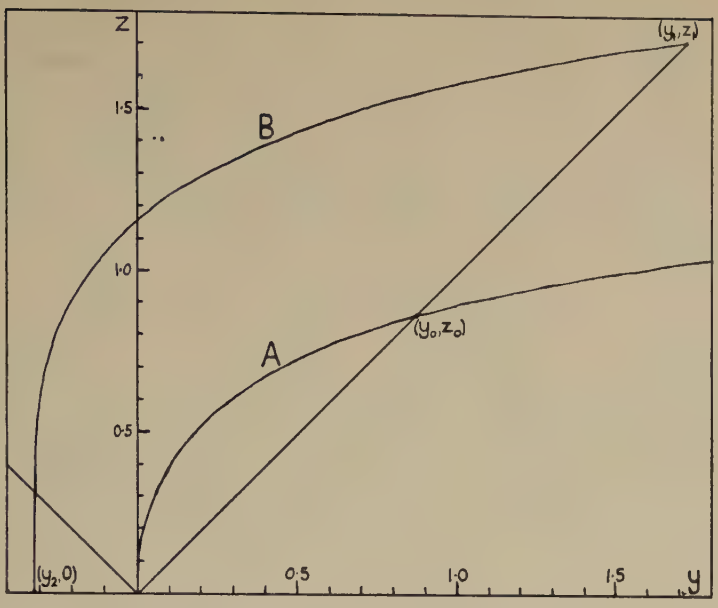


Figure 18. Solution of the charging equations.

Now, from (12.19) and (12.20) we have

$$\frac{Q^*}{a^*} = \frac{y_2}{{z_0}^2}\frac{3\pi}{4k} J x_0, \quad . \quad . \quad . \quad . \quad (12.26)$$

thus, for $n = 1$,

$$Q^*/a^* = -0{\cdot}32\,\pi J x_0/k, \quad . \quad . \quad . \quad (12.27)$$

which differs from the corresponding equation of the previous case only in the numerical value of the constant. As before this relation is insensitive to variation in n; for $n = 0{\cdot}25$, $z_0 = 0{\cdot}395$ and $y_2 = -0{\cdot}097$ giving $y_2/{z_0}^2 = -0{\cdot}62$, while for $n = 4{\cdot}0$, $z_0 = 1{\cdot}31$ and $y_2 = -0{\cdot}555$, giving $y_2/{z_0}^2 = -0{\cdot}32$.

Substituting for k and rearranging, we have, finally,

$$J x_0 = -7{\cdot}5 \times 10^3\, Q^*/a^* \quad . \quad . \quad . \quad (12.28)$$

Before proceeding to compare these two cases with the observations we shall consider briefly the assumptions and approximations of the analysis.

(*a*) *Initial charges.* Eqs. (12.7) and (12.24) were taken to represent the initial charge-size relation for mathematical simplicity. However, small deviations from this relation do not appreciably affect the relations between Q^*/a^* and Jx_0 since it has been shown that these are insensitive to the values of the initial charges. Conversely, little information can be obtained from the observations about the initial charges.

(*b*) *Variation of current density with height.* The current density changes with height from two main causes. First, there is an increase with height as the cloud is not of infinite lateral extent. The magnitude of this effect cannot at present be estimated. Second, the capture of ions by the drops results in a decrease of current density with height. Observations by Simpson (1949) and computations by Chalmers (1951) indicate a reduction by about 30 per cent for a rainfall rate of 10 mm/hr.

(*c*) *Charging within the cloud.* The ions of the discharge current are captured by cloud droplets within a few metres inside the cloud and are carried upwards. Thus the

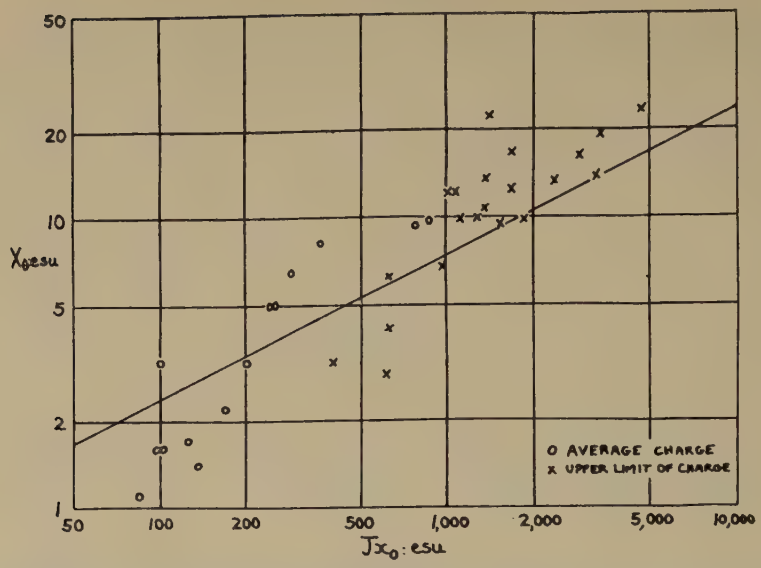


Figure 19. The field in the charging region plotted against Jx_0, logarithmic scale. The line is the theoretical relation $X = (8\pi Jx_0/w)^{\frac{1}{2}}$, for $w = 450$ esu.

capture of charge may begin some distance within the cloud. The equation of charging from this cause within the cloud is

$$\frac{dQ}{dz} = \frac{\pi J a^2}{v_a}, \qquad . \quad . \quad . \quad . \quad (12.29)$$

assuming that the drop captures all the droplets in the volume swept out. The initial rate of charging below the cloud, obtained by putting $Q = 3Xa^2$ in (12.4) is

$$\frac{dQ}{dx} = \frac{12\pi J a^2}{v_a} \qquad . \quad . \quad . \quad . \quad (12.30)$$

Thus the rate of charging increases by a factor of 12 as the drops leave the cloud. This means that the height x_0 is somewhat greater than the height of the cloud base.

(*d*) *The nature of the particles.* On some occasions the drops may fall part of the way to the ground as snow or ice. The process of charging applies equally to these particles (Wormell 1948) again resulting in (12.12) and (12.27) although the appropriate value of k is smaller by a factor of about five.

13. Comparison with observations

(*a*) *Variation of field with height*

We have seen, in the previous section, how, if we assume that the field has a constant value X_0 in the region below the cloud, we may obtain an approximate value from the observed values of Q^* and a^* by using (12.13). The resulting values of X_0 for the present series of observations range from 1·1 esu to 9·8 esu (330 v/cm to 3,000 v/cm) and exceed the corresponding value of the field at the ground (X_g, Table 2) by factors ranging from 4·6 to 98, the mean value being 30. It appears therefore that the field in the charging region is greater than the field at the ground by at least an order of magnitude.

In Fig. 19 these values of X_0 are plotted against the corresponding value of the product Jx_0 obtained from the observations, using (12.16). Points for the upper limit of charge obtained similarly from Q' and a' are also plotted. A logarithmic scale is used and signs are not differentiated. The line on the graph is (12.5), with $w = 450$ ESU, which relates the field at a height x_0 to the product Jx_0. The points for the separate distributions are scattered but show general agreement with the theoretical relation, thus supporting the existence of the space charge of ions from point discharge.

(*b*) *The charge-size relation*

We have already seen that there is qualitative agreement between the observed and computed charge-size relations. We may use (12.28) to derive, from the observed values of Q^* and a^*, a value of $(Jx_0)^*$ which can then be compared with the value obtained independently from the product of point-discharge current density and the estimated height of the cloud base. In Table 3 the values of Jx_0 derived by the two methods are compared and it is seen that the value derived from the measured charges on the drops is, in general, greater than that obtained directly, at the ground, by a factor of about 6. There is some uncertainty in the calibration of the point-discharge current but this factor appears to indicate an increase of current density with height. It is considered that the interpretation is supported by this comparison.

TABLE 3. COMPARISON OF DIRECT AND INDIRECT VALUES OF Jx_0.

Distribution number	$J \times 10^3$ ESU	$x_0 \times 10^{-3}$ cm	Jx_0 ESU	$(Jx_0)^*$ ESU
1	2·0	15	30	150
7A	− 2·9	30	− 87	− 535
22	1·5	30	45	190
23A	− 2·4	45	− 108	− 250
24A	− 1·1	30	− 33	− 150
25	2·9	45	130	125
29A	0·5	45	23	1290
30A	− 1·1	45	− 50	− 375
33	− 0·7	45	− 32	− 300

(*c*) *The range of charge*

Some indication of the cause of the large range of charge which is characteristic of all the observations may be obtained from a comparison of the observed distributions with the computed charge-size relations. Variation in either Jx_0 or n, or both, could lead to a large range of charges on the larger drops; however, for drops smaller than 0·5 mm diameter, only variation in Jx_0 is sufficient to produce a range of charge which includes both signs.

It is reasonable to assume that x_0, the initial height, is the same for drops with the average charge as for drops with the upper limit of charge. Thus the ratio $(Jx_0)'/(Jx_0)^*$, where $(Jx_0)'$ is the value of Jx_0 derived from Q' and a', gives the ratio between the maximum current density at the cloud base and the mean taken over the area through which drops are leaving the cloud. For the thirteen distributions for which sufficient data are available the ratio varies between 3·0 and 10·0, with a mean value of 5·5. It appears, therefore, that the range of charge between the average charge and the upper limit of charge can be explained adequately by variation in current density.

Similarly, we may interpret the lower limit of charge as the charge on the drops which have fallen through the region of minimum current density. Drops falling through this region will have experienced little change in charge and the charge-size relation at the ground will be nearly the same as the initial charge-size relation. Therefore, since the limits of charge for the smallest drops are roughly symmetrical we have justification for the assumption that n is about equal to unity.

It remains to be shown that lateral mixing of the drops, while falling, is sufficient to bring together at the ground drops originating in different areas of the cloud. Observations by Davies (1950), for example, have shown that this is possible if the initial separation is not greater than about 100 m. If this represents the size of the charged region in the cloud it differs considerably from Wilson's (1920) estimate of 1 km.

It was noted earlier that the range of charge on the largest drops in a distribution is small (Fig. 14) and that the charges on these drops correspond to what is the upper limit of charge on smaller drops. This means that the larger drops are not diffused away from the region of high current density to the same extent as the smaller drops. This can also be seen in Fig. 13 where the smaller drops are concentrated towards the lower limit of charge while the larger drops are concentrated towards the upper limit of charge.

Several of the distributions contain a small group of drops which fall outside the main part of the distribution and yet, within the group, show a charge-size relation which is similar to that of the adjacent upper limit of charge. Well marked examples of these secondary limits occur in distributions number 8B, 23C, 29A and 30A; there are less obvious examples in distributions number 8E, 22 and 30E. These groups of drops each correspond to a value of Jx_0 which is greater than that of the respective upper limit of charge. It is not clear why the range of charge is not continuous up to this maximum value.

14. Conclusions

It has been shown in the previous sections that the charge-size relation which is observed on drops at the ground is largely determined by the capture of ions from the point-discharge current. Discrepancies between theory and experiment can be ascribed to the use of a simple model to represent such a complex phenomenon as an electrified cloud and its environment.

A feature of interest is the remarkable stability shown by some of the clouds, particularly those of observation numbers 8 and 30. The distributions show only slight changes during these two periods of observation which were 24 and 13 min respectively. On other occasions there was not this stability. The sudden increase of the range of charge which occurred during the first five minutes of observation number 23, Fig. 12b, is interpreted as the development of the electrical dipole within the cloud. The instability of this particular shower cloud is also seen in the record of rate of rainfall, Fig. 6a. The rate increased rapidly to a maximum of 37 mm/hr, then decreased equally rapidly, whereas, during observations 8 and 30, the rate of rainfall was relatively steady at about 3 mm/hr.

Another feature which this investigation appears to indicate is the occurrence of clouds of negative polarity. Negative distributions, which imply clouds of negative polarity, were observed on 5 occasions (numbers 1, 22, 25, 28 and 34) and positive distributions on 9 occasions (numbers 6, 7, 8, 14, 15, 23, 24, 30 and 33); during one observation (number 29) the distributions changed sign, the first two being negative and the third positive.

There are few other data available concerning the polarity of non-thundery clouds. For thunderclouds, however, investigations by various methods have indicated positive

polarity almost without exception. Thus it must be concluded that, although a cloud may become electrified with either positive or negative polarity, it is only in cases of positive polarity that the electrical activity can develop sufficiently to result in a lightning discharge.

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The influence of variable mixing ratio and temperature on the radiation flux

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Summary

A rapidly convergent solution is obtained for the equations of radiative transfer. An absorbing atmosphere having an arbitrary variation of mixing ratio, temperature and pressure with height is considered. The Lorentz shape of the spectral lines together with the dependence of their line strengths and half-widths on pressure and temperature are taken into account in the calculations. It is assumed that the lines do not overlap and the range of validity of this approximation is stated. It is shown that the expression for the absorption of radiation over a path between any two heights can be written in a tractable form even after three-fold integration over the variable conditions along the path, over frequency and over all angles in the hemisphere. When the mass per unit area, along the path, is small or the line intensity is small or both, thus causing the absorption of radiation to be small even at the centre of the line, the integrated absorption is proportional to the dimensionless constant gamma that occurs in atmospheric radiation problems; when these quantities are large, thus causing the absorption to be complete for several half-widths from the centre of the line, the integrated absorption is proportional to the square-root of gamma. This is the generalization to variable path conditions of the linear and square-root relations of infra-red spectroscopy. Some examples of the method developed here are given when the variable conditions along the atmospheric path can be represented with sufficient accuracy as polynomials. An approximation to the H_2O and CO_2 distributions shows that the absorption of radiation from the ground at a given height depends greatly on the variation of the mixing ratio with height.

1. Introduction

The complexity of atmospheric radiation calculations has been appreciated by meteorologists for more than a half century. Because of the extremely rapid variation of the absorption coefficient with frequency, it seemed necessary to make drastically simplifying assumptions in order to obtain a solution. Thus the first solutions obtained for the radiation flux assumed a grey atmosphere. In this approximation the dependence of the absorption coefficient on frequency, pressure, and temperature is either neglected or taken into account by a rough averaging process. A major objection to this procedure is that it has not been possible to state the magnitude of the error in the final answer due to these many approximations.

Furthermore, the grey-atmosphere approximations do not yield as simple final results as had been expected originally. It is quite remarkable that the final answers are no more complicated when the Lorentz line-shape is correctly taken into account, together with the dependence of the half-width and intensity of the spectral lines, and of the mixing ratio, on the height in the atmosphere, than for the grey-atmosphere approximation. Even after the three-fold integration over the path length, over frequency and over the angles in the hemisphere, the result is still in a tractable form. The first model where the variation of all these parameters was correctly taken into account was that of a band of non-overlapping lines of a gas with constant mixing ratio in an isothermal atmosphere and was derived by Pedersen (1942) and Strong and Plass (1950). Additional models have since been investigated to include certain cases where the atmosphere is non-isothermal, where the lines of the band overlap appreciably, where the mixing ratio varies with height, and where the Doppler effect contributes to the half-width by Kaplan (1952), King (1952a, 1952b), Plass (1952a, 1952b, 1954), and Plass and Fivel (1953).

Two of the three principal gases that contribute to the infra-red radiation flux, water vapour and ozone, have mixing ratios that vary greatly with height , and exist in the atmosphere over a wide range of temperatures. The intensity of an infra-red absorption line

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is usually a rapidly varying function of temperature and this change in intensity influences the radiation flux. Yet none of the above papers has shown whether these variations are important for meteorological problems. Here a generalized method for the calculation of the radiation flux is developed with the particular aim of finding the influence of the mixing ratio and temperature variations with height. The result is expressed as a rapidly convergent series in certain moments of the atmospheric distribution and includes as special cases all of the results that have been derived previously for non-overlapping lines. The calculation is designed so that the particular pressure and temperature variation with height found in a given sounding can be used to calculate the radiation flux.

The derivation that is given here can apparently be stated in a useful form only for the case of non-overlapping spectral lines. However, this restriction does not limit the range of validity of these results as much as has frequently been implied in the literature. The regions of validity of the single-line approximation have been given by Plass (1954). For example, a comparison of the equations for the Elsasser model of a band with those for a non-overlapping band shows that the latter equations are correct to within 5 per cent when $\gamma \leqslant 60 \cos\theta$ for $\beta = 0{\cdot}1$; when $\gamma \leqslant 6{,}000 \cos\theta$ for $\beta = 0{\cdot}01$; when $\gamma \leqslant 600{,}000 \cos\theta$ for $\beta = 0{\cdot}001$, where $\beta = 2\pi\alpha/d$, α is the half-width of the lines, d is their spacing and γ is defined by Eq. (7). Since the most recent measurements indicate that $\beta \simeq 0{\cdot}3$ at atmospheric pressure for CO_2 and H_2O, the equations derived from the single line model are accurate within 5 per cent everywhere in the stratosphere when $\gamma \leqslant 125 \cos\theta$. This would include all but a few of the very strongest lines in both bands.

An actual band may not be accurately represented by the Elsasser model or any other simple model. An analysis of the experimental data for CO_2 by the method of Plass (1952a), however, leads to the same general results for the range of validity as are given above. This method is valid for an arbitrary variation of the spacing and intensity of the spectral lines in a band and the only restriction is that the path length be greater than a certain amount. From a plot of the experimental results, this minimum path length can easily be determined. The preliminary analysis of the CO_2 data shows that the single-line model is accurate to within 5 per cent for CO_2 when the fractional absorption is less than 0·80, 0·65 and 0·30 for the ranges 14-16 μ, 13-14 and 16-17 μ, and 12-13 and 17-18 μ respectively. Since these results are independent of any particular model that is assumed for the band, we conclude that for many problems there are important frequency ranges that can be treated by the single-line equation. Except for a few of the strongest lines in a band this approximation may be used everywhere in the stratosphere.

Meteorologists will probably be most interested in the results given in section 5 and Figs. 1-3. As an example, Fig. 2 shows that the radiation from the ground absorbed by equal total masses of CO_2 and H_2O and by spectral lines of the same intensity decreases much more rapidly with height for H_2O than it does for CO_2. Thus these numerical results show the importance of including the vertical variation of mixing ratio with height in radiation calculations.

2. Evaluation of transmission function

The fundamental problem in the calculation of infra-red radiation fluxes is the evaluation of the integrated absorption, $A(u_0, u_1)$, defined as

$$A(u_0, u_1) = \int_{\Delta\nu} \left\{1 - \exp\left[-\sec\theta \int_{u_0}^{u_1} k(\nu, u)\, du\right]\right\} d\nu, \quad . \quad . \quad (1)$$

where $\Delta\nu$ is the frequency interval, θ is the angle that the radiation makes with the

vertical, and $k(\nu, u)$ is the absorption coefficient. The mass per unit area, u, is defined by the equation

$$u = \int_z^\infty c\,\rho\,dz = \int_z^\infty \rho_T\,dz, \qquad . \quad . \quad . \quad (2)$$

where the fractional concentration (mixing ratio) c is the ratio of the density of the radiating gas, ρ_T, to the total density, ρ. The transmission function between two different heights is specified by the mass per unit area, u_0 and u_1, at these heights. The transmission function given by (1) is a solution of the equation of radiative transfer. The radiative flux at a given height or the absorption of radiation at a given height can be calculated very simply if the transmission function for a given frequency interval is known (see, for example, Plass (1952b), Plass and Fivel (1953)). Thus the primary problem is the calculation of the transmission function.

In order to evaluate the transmission function (1), it is assumed that the lines have the Lorentz-shape

$$k(\nu) = (S\alpha/\pi)\left[(\nu - \nu_0)^2 + \alpha^2\right]^{-1}, \qquad . \quad . \quad . \quad (3)$$

where S is the line intensity and α is the half-width of the lines. This assumption introduces no appreciable errors up to heights of about 50 km, as has been shown by Plass and Fivel (1953). Recent theoretical and experimental evidence also indicates that possible asymmetries and shift of the line centre with pressure are unimportant for atmospheric radiation problems.

The half-width, α, is a function of both pressure and temperature, which according to kinetic theory, is

$$\alpha = \alpha_s\,(p/p_s)\,(T_s/T)^{\frac{1}{2}}, \qquad . \quad . \quad . \quad . \quad (4)$$

where the subscript s refers to an arbitrary standard state. The pressure dependence of (4) has now been verified both experimentally and theoretically over the range of pressures of interest in meteorological applications. There still appears to be a slight doubt about the temperature factor in (4), but in any case it gives only a very minor correction to the results derived below. If the temperature dependence should be different from that given in (4) for a given case, the corresponding correction can be made very easily in the following equations. Of much greater importance is the temperature variation of the line intensity, S, which may change by several orders of magnitude for the temperature range that occurs in the atmosphere.

The procedure that is developed in this paper to evaluate (1) may seem rather formal. The usefulness of the final result is the main justification for this particular expansion. The extremely rapid convergence of the series in the final result is due to the method of expansion which converges rapidly in all the important physical limits, i.e., for strong and weak lines and for short and long path-lengths at each step in the calculation. It is instructive to compare each step of the general calculation with the particular example of an isothermal atmosphere with constant mixing ratio.

In order to evaluate (1), we take the half-width, α, as the independent variable. At a given height in the atmosphere there is a unique value of α and all other variables are considered as a function of α. (In any actual atmospheric problem the very much more rapid variation of p than of T with height and the dependence given by (4) guarantees the uniqueness of α as a function of height. If in some unusual problem, the height should turn out to be a multiple-valued function of α, it would be necessary to perform the calculation given below separately for each of several regions in which it could be taken to be single-valued). For example, the line intensity S is a temperature-dependent variable. However we can write $S = S(\alpha)$, since the temperature at a given height is

known for a particular sounding and the half-width, α, appropriate to that height can be calculated from (4).

Since $gdu = cdp$ and since from (4)

$$dp = \frac{p_s T^{\frac{1}{2}}}{\alpha_s T_s^{\frac{1}{2}}} \left(1 + \tfrac{1}{2} \frac{d(\ln T)}{d(\ln \alpha)}\right) d\alpha, \quad . \quad . \quad . \quad (5)$$

the transmission function (1) for a single line can be written in the form

$$A(\alpha_0, \alpha_1) = \int_{-\infty}^{\infty} \left\{1 - \exp\left[-\gamma_s \int_{\alpha_0}^{\alpha_1} G(\alpha) (\nu^2 + \alpha^2)^{-1} d\alpha\right]\right\} d\nu, \quad . \quad (6)$$

where

$$\gamma_s = S(\alpha_s)\, c(\alpha_s)\, p_s \sec\theta / 2\pi\alpha_s g, \quad . \quad . \quad . \quad (7)$$

$$G(\alpha) = \frac{2 S(\alpha) c(\alpha) T^{\frac{1}{2}}(\alpha)}{S(\alpha_s) c(\alpha_s) T^{\frac{1}{2}}(\alpha_s)} \left(1 + \tfrac{1}{2} \frac{d(\ln T)}{d(\ln \alpha)}\right) \alpha \simeq \frac{2 S(\alpha) c(\alpha) \alpha}{S(\alpha_s) c(\alpha_s)}, \quad . \quad (8)$$

where the zero of frequency has been shifted to the centre of the line and where s refers to the values of the variables at some standard reference level that can be chosen arbitrarily at the beginning of the calculation.

The terms in (8) which explicitly involve the temperature are usually a small correction. Since this combination of factors occurs several times in this work, we evaluate it here for the case $T = T_0 - az$. The dependence of pressure on height is then

$$p = p_0 \left(1 - \frac{az}{T_0}\right)^{Mg/aR}$$

Thus we obtain from (4) that $d(\ln T)/d(\ln \alpha) = [(Mg/aR) - \frac{1}{2}]^{-1}$.

For an average temperature gradient in the troposphere

$$\frac{T^{\frac{1}{2}}(\alpha)}{T^{\frac{1}{2}}(\alpha_s)} \left(1 + \tfrac{1}{2} \frac{d(\ln T)}{d(\ln \alpha)}\right) \simeq 1{\cdot}12 \frac{T^{\frac{1}{2}}(\alpha)}{T^{\frac{1}{2}}(\alpha)_s}.$$

Therefore this factor is only a 12 per cent correction to $G(\alpha)$ in the troposphere when there is a constant temperature gradient.

The dimensionless constant γ_s given by (7) occurs in all radiation problems where the Lorentz line-shape is taken into account, as was shown by Strong and Plass (1950). Plass (1954) has shown in detail that this constant γ_s for atmospheric problems is analogous to the dimensionless constant $x = Su/2\pi\alpha$, which occurs in the expressions for absorption observed under constant pressure conditions along the entire path in the laboratory. In the case of a gas with variable fractional concentration, the value of γ_s depends on the standard level chosen. In the expression for any result that can be measured physically, γ_s must occur in combination with other factors so that the final result is independent of the standard level chosen.

The function $G(\alpha)$ defined by (8) can be calculated for a given atmospheric sounding specifying the temperature and pressure as a function of height from (4) and tables of the variation in line intensity with temperature. The latter can be calculated from the Boltzmann factor if the energy levels are known. The necessary data for such calculations are being published, e.g., Kaplan (1952). In many problems it may be possible to represent $G(\alpha)$ by a polynomial in α with sufficient accuracy. In this case the procedure developed here is very easy to solve, as discussed in Section 5. For the remainder of this paper the function $G(\alpha)$ is considered as a known function of α. This allows us to solve (6) for the transmission function.

We may write the integral in (6) as the following double integral

$$A(\alpha_0, \alpha_1) = 2 \int_0^{\infty} d\nu \int_0^{y(\nu)} e^{-y}\, dy, \qquad (9)$$

where the integration over ν is taken from 0 to ∞ only and the value of the integral is multiplied by two, since the integrand is an even function of ν and where

$$y(\nu) = \gamma_s \int_{\alpha_0}^{\alpha_1} G(\alpha)\, (\nu^2 + \alpha^2)^{-1}\, d\alpha. \qquad (10)$$

In order to reverse the order of the double integration in (9), (10) must be solved to obtain $\nu = \nu(y)$. Here it is necessary to have a function which, together with its derivatives, is finite at the origin. Such a function is $\psi(y)$, which is defined as

$$\psi(y) = y^{\frac{1}{2}}\, \nu(y). \qquad (11)$$

Also the moments, μ_l, of the distribution of the radiating gas are defined as

$$\mu_l = \int_{\alpha_0}^{\alpha_1} \alpha^{2l}\, G(\alpha)\, d\alpha. \qquad (12)$$

Then (9) may be written as

$$A(\alpha_0, \alpha_1) = 2 \int_0^{\gamma_s \mu_{-1}} dy\, e^{-y} \int_0^{\nu(y)} d\nu$$

$$= 2 \int_0^{\gamma_s \mu_{-1}} \psi(y)\, y^{-\frac{1}{2}}\, e^{-y}\, dy. \qquad (13)$$

Expand $\psi(y)$ in a power series in y to obtain

$$\psi(y) = \sum_{n=0}^{\infty} \frac{\psi^{(n)}(0)}{n!}\, y^n. \qquad (14)$$

Next we determine the value of $\psi^{(n)}(0)$, the n-th derivative of ψ with respect to y evaluated at $y = 0$. From (10) and with the use of (11), we may write

$$\psi^2(y) = \gamma_s \int_{\alpha_0}^{\alpha_1} G(\alpha)\, [1 + \alpha^2\, \psi^{-2}(y)\, y]^{-1}\, d\alpha. \qquad (15)$$

Setting $y = 0$, we immediately obtain that

$$\psi(o) = \gamma_s^{\frac{1}{2}}\, \mu_0^{\frac{1}{2}}. \qquad (16)$$

By performing n differentiations of (15), we find the value of $\psi^{(n)}(0)$ to be

$$\psi^{(1)}(0) = -\mu_1/2\, \gamma_s^{\frac{1}{2}}\, \mu_0^{\frac{3}{2}}$$

$$\psi^{(2)}(0) = (4\, \mu_0\, \mu_2 - 5\, \mu_1^2)/4\, \gamma_s^{\frac{3}{2}}\, \mu_0^{\frac{7}{2}} \qquad (17)$$

$$\psi^{(3)}(0) = -(24\, \mu_0^2\, \mu_3 - 84\, \mu_0\, \mu_1\, \mu_2 + 63\, \mu_1^3)/8\, \gamma_s^{\frac{5}{2}}\, \mu_0^{\frac{11}{2}}$$

. . . . .

. . . . .

. . . . .

When (14) is substituted in (13), the result is obtained that

$$A(\alpha_0, \alpha_1) = 2 \sum_{n=0}^{\infty} \frac{\psi^{(n)}(0)}{n!}\, \Gamma(n + \tfrac{1}{2}, \gamma_s \mu_{-1}), \qquad (18)$$

where the incomplete gamma function

$$\Gamma(a, b) = \int_0^b x^{a-1}\, e^{-x}\, dx. \qquad (19)$$

order to show explicitly the dependence of (18) on γ_s and μ_0, define the constants to be equal to the numerators of $\psi^{(n)}(0)$, that is

$$\lambda_n = 2^n \gamma_s^{n-\frac{1}{2}} \mu_0^{2n-\frac{1}{2}} \psi^{(n)}(0). \quad . \quad . \quad . \quad (20)$$

The resulting expression for the transmission function is

$$A(\alpha_0, \alpha_1) = \sum_{n=0}^{\infty} \frac{\lambda_n}{2^{n-1}\, n!\, \mu_0^{2n-\frac{1}{2}} \gamma_s^{n-\frac{1}{2}}} \Gamma(n + \tfrac{1}{2}, \gamma_s \mu_{-1}), \quad . \quad . \quad (21)$$

here

$$\lambda_0 = 1;\ \lambda_1 = -\mu_1;\ \lambda_2 = 4\mu_2 \mu_0 - 5\mu_1^2; \quad . \quad . \quad (22)$$

Eq. (21) is the desired generalized expression for the absorption between two rbitrary heights integrated over the path between them and over frequency. For any iven distribution of pressure and temperature with height it is possible to compute ll the coefficients in (21). In practice it is found that the infinite series in (21) is very apidly convergent and that in most cases the first term alone or at most the first few erms give an answer to the required accuracy, as shown by the examples considered in he following sections. This result is for beam radiation that makes an angle θ with the ertical, since γ_s from (7) is a function of this angle θ. The integration of (21) over the emisphere in order to obtain the diffuse radiation is done in Section 6.

3. Asymptotic behaviour

The transmission function (21) can be further simplified, for an atmosphere which as an arbitrary composition and distribution of pressure and temperature with height, or the important limiting cases of large and small absorption near the centre of the line. The following expansions of the incomplete gamma function (see, e.g., Erdély (1953) Vol. II, p. 135) are needed to study this limiting behaviour :

$$\Gamma(a, b) = \sum_{m=0}^{\infty} \frac{(-1)^m b^{a+m}}{(a + m)\, m!} \quad . \quad . \quad . \quad (23)$$

nd

$$\Gamma(a, b) = \Gamma(a) - b^{a-1} e^{-b} \sum_{m=0}^{M} \frac{\Gamma(1 - a + m)}{\Gamma(1 - a)(-b)^m}, \quad . \quad . \quad (24)$$

where $\Gamma(a)$ is the gamma function.

(a) *Absorption small, even at centre of line.*

A case of considerable practical importance occurs when the absorption is small, ven at frequencies near the centre of the line. This can occur for two physically distinct ituations. For the transmission between two given heights, there always are sufficiently eak lines (small γ_s) for this condition to be satisfied. On the other hand, for a given ne, no matter how strong it may be, this condition can always be satisfied for the trans- ission between two points sufficiently close together.

The mathematical condition that must be satisfied in all the above cases is that

$$\gamma_s \mu_{-1} = (p_s \sec\theta / \pi\, \alpha_s\, g) \int_{\alpha_0}^{\alpha_1} S(\alpha)\, c(\alpha)\, \alpha^{-1}\, d\alpha \ll 1.$$

f this condition is satisfied, we obtain from (21) and (23) that

$$A(\alpha_0, \alpha_1) = \gamma_s \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{(-1)^m \mu_{-1}^{n+m+\frac{1}{2}} \lambda_n \gamma_s^m}{2^{n-1}\, n!\, m!\, (n + m + \frac{1}{2})\, \mu_0^{2n-\frac{1}{2}}}. \quad . \quad . \quad (25)$$

Since (25) is a convergent series in ascending powers of γ_s, whenever there ar sufficiently weak lines or when the path length is sufficiently small, so that $\gamma_s \mu_{-1} \ll$ the higher terms in the series can be neglected for sufficiently small values of the para meters. Under these conditions *the integrated absorption varies linearly as a function of* regardless of how the mixing ratio, temperature, or pressure changes with height. well-known result of infra-red spectroscopy states that, when there are constant pressur and temperature conditions along the entire path, the integrated absorption is a linea function of $x = Su/2\pi\alpha$, when x is small. Thus (25) is the appropriate generalizatio of this result to variable atmospheric conditions.

If the path length is sufficiently small, we may write $\alpha_1 = \alpha_0 (1 + \epsilon)$, where $\epsilon \ll 1$ Even for an arbitrarily large line strength S, if the path length is sufficiently small, the the absorption is small even at the line centre and (25) is valid. Thus A is a linear functio of γ_s. However, in this interesting and important case the summation over n in (25 can be evaluated explicitly. In this limit from (12), μ_{-1} and μ_0 are immediately found to b

$$\mu_{-1} = \epsilon \alpha_0^{-1} G(\alpha_0),$$

$$\mu_0 = \epsilon \alpha_0 G(\alpha_0). \quad . \quad . \quad . \quad . \quad (26$$

From (15), we have

$$\psi(y) = [\gamma_s \epsilon \alpha_0 G(\alpha_0) - \alpha_0^2 y]^{\frac{1}{2}}. \quad . \quad . \quad . \quad (27$$

Evaluating the nth derivative of ψ at $y = 0$ and using (20), we find that

$$\lambda_n = -\frac{(2n)!}{2^n (n)! (2n-1)} \epsilon^n G^n(\alpha_0) \alpha_0^{3n}. \quad . \quad . \quad (28$$

Substituting (26) and (28) into (25) and rearranging the terms slightly, the result is obtaine that

$$A(\alpha_0, \alpha_1) = \gamma_s \sum_{m=0}^{\infty} \frac{(-1)^{m+1} \gamma_s^m \epsilon^{m+1} G^{m+1}(\alpha_0)}{(m+1)! \alpha_0^m} \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n} (n!)^2} \left[\frac{1}{n - \frac{1}{2}} - \frac{1}{n + m + \frac{1}{2}}\right]. \quad (29$$

The summation over n can be evaluated in terms of the beta function,

$$B(p, q) = \frac{\Gamma(p)\,\Gamma(q)}{\Gamma(p+q)} = \int_0^1 x^{p-1} (1-x)^{q-1}\, dx,$$

since (e.g., Whittaker and Watson (1927), p. 259)

$$B(p, \tfrac{1}{2}) = \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n} (n!)^2} \frac{1}{n+p}. \quad . \quad . \quad . \quad . \quad (30$$

It is found that the sum of the first term inside the square brackets in (29) is zero and tha the second term gives

$$A(\alpha_0, \alpha_1) = \gamma_s \epsilon G(\alpha_0) \sum_{m=0}^{\infty} \frac{(-1)^m \gamma_s^m \epsilon^m G^m(\alpha_0)}{(m+1)! \alpha_0^m} B(m + \tfrac{1}{2}, \tfrac{1}{2}) \quad . \quad (31$$

$$= \pi \gamma_s \epsilon G(\alpha_0) \left[1 - \frac{\gamma_s \epsilon G(\alpha_0)}{4 \alpha_0} + \ldots\right].$$

The beta functions of half-integral arguments that occur in (31) are, of course, known numbers.

From the condition $\gamma_s \mu_{-1} \ll 1$ for the validity of (25) and from (26), it is seen that (31) is a rapidly convergent expression for the integrated absorption for a line o *any* strength, when $\alpha_1 - \alpha_0 \ll \gamma_s \alpha_0^{-2} G(\alpha_0)$. Thus (31) gives the integrated absorptio for a line of any strength provided the path length is sufficiently short.

(*b*) *Absorption complete for several half-widths from line centre.*

Another important limit for the general expression for the absorption integrated over frequency as given by (21), occurs when the absorption is virtually complete for several half-widths from the line centre. The mathematical condition for this limit is that $\gamma_s \mu_{-1} \gg 1$. Substituting (24) in (21), we have that

$$A(\alpha_0, \alpha_1) = \sum_{n=0}^{\infty} \frac{\lambda_n}{2^{n-1} n! \, \mu_0^{2n-\frac{1}{2}} \gamma_s^{n-\frac{1}{2}}}$$

$$\times \left[\Gamma(n + \tfrac{1}{2}) - (\gamma_s \mu_{-1})^{n-\frac{1}{2}} \exp(-\gamma_s \mu_{-1}) \sum_{n=0}^{M} \frac{(\frac{1}{2} - n + m)}{\Gamma(\frac{1}{2} - n)(-\gamma_s \mu_{-1})^m} \right]. \quad . \quad (32)$$

As can be shown by using Stirling's approximation for the factorial, all the terms, except the first, in the square brackets above may be neglected when $\gamma_s \mu_{-1} \gg n$. Since the summation over n in (32) converges rapidly in all cases of interest (usually only one or two terms need be used), an approximate evaluation of (32) may be obtained by retaining only the $\Gamma(n + \frac{1}{2})$ term in the square bracket. This correctly evaluates the terms in the series from $n = 0$ to $n \sim \gamma_s \mu_{-1}$. For values of n such that $n \gg \gamma_s \mu_{-1}$, this approximation overestimates the value of the square bracket in (32), since it is always true that $\Gamma(a) \geqslant \Gamma(a, b)$. Thus, if the terms for $n \gg \gamma_s \mu_{-1}$ are negligible in this approximate expression, they are even smaller in the actual expression and this approximation correctly evaluates (32). Thus when $\gamma_s \mu_{-1} \gg 1$, we may write (32) as

$$A(\alpha_0, \alpha_1) = \pi^{\frac{1}{2}} \gamma_s^{\frac{1}{2}} \mu_0^{\frac{1}{2}} \sum_{n=0}^{\infty} \frac{(2n)! \, \lambda_n}{2^{3n-1} (n!)^2 \mu_0^{2n} \gamma_s^n}. \quad . \quad . \quad (33)$$

The terms for $n \gg \gamma_s \mu_{-1}$ are small and only the first few terms are important in (33) when $\gamma_s \mu_{-1} \gg 1$. When this inequality is satisfied, the series in (33) converges very rapidly. Under these conditions *the integrated absorption varies as the square root of the parameter* γ_s regardless of how the mixing ratio, temperature, or pressure changes as a function of height. This is the generalization to the variable path conditions that occur in the atmosphere of the well-known result that, for constant pressure and temperature along the entire path, the integrated absorption varies as the square-root of $x = Su/2\pi\alpha$, when x is large. In the atmosphere the relevant condition is that $\gamma_s \mu_{-1} \gg 1$. This is satisfied for a given path by all lines with strengths greater than a certain critical amount or it is satisfied for any given line by taking a sufficiently long path length.

As the line strength or path length is increased still further, the lines in an actual band begin to overlap. On the basis of the Elsasser model of a band with lines that have equal spacing and equal intensity, it can be shown by a method similar to the one used by Plass (1954) that the overlapping changes $A(\alpha_0, \alpha_1)$ appreciably only when $\gamma_s \mu_{-1} \gg (d/2\pi\alpha_1)^2$, where d is the spacing between the lines. As the overlapping increases, $A(\alpha_0, \alpha_1)$ increases less rapidly than $\gamma_s^{\frac{1}{2}}$ and finally approaches the maximum possible value of unity for a frequency interval with complete absorption at all frequencies.

4. Isothermal atmosphere with constant mixing ratio

The equations derived above can be simplified considerably for the important case of an isothermal atmosphere with a constant mixing ratio. This is the simplest illustration of the methods developed in the preceding section. For any radiating gas, the approximation of a constant mixing ratio and isothermal conditions is adequate for sufficiently short path lengths. For gases, such as CO_2, that are uniformly mixed to great heights, it is an

adequate approximation over quite long path lengths (especially when the temperature variation is taken into account by other techniques). Although the results for this special case have been derived and discussed in detail by Strong and Plass (1950), Plass (1952b, 1954), Kaplan (1952) and King (1952a, 1952b), we present them again in this section. None of these authors has considered the expression for the integrated absorption in all the possible limits between two arbitrary heights in the atmosphere. This general expression is given here.

For an isothermal atmosphere with constant mixing ratio, from (8) and (12) we have that

$$G(\alpha) = 2\alpha,$$

$$\mu_{-1} = 2 \ln (\alpha_1/\alpha_0), \quad . \quad . \quad . \quad . \quad (34)$$

$$\mu_n = (n+1)^{-1} (\alpha_1^{2n+2} - \alpha_0^{2n+2}), \quad n \neq -1.$$

Thus from (21), using the values of λ_n calculated from (22), we find that

$$A(\alpha_0, \alpha_1) = 2\gamma_s^{\frac{1}{2}} (\alpha_1^2 - \alpha_0^2)^{\frac{1}{2}} \left\{ \Gamma(\tfrac{1}{2}, \gamma_s \mu_{-1}) - \frac{\alpha_1^2 + \alpha_0^2}{4\gamma_s (\alpha_1^2 - \alpha_0^2)} \Gamma(\tfrac{3}{2}, \gamma_s \mu_{-1}) \right.$$

$$+ \frac{\alpha_1^4 - 14\alpha_1^2\alpha_0^2 + \alpha_0^4}{96\gamma_s^2 (\alpha_1^2 - \alpha_0^2)^2} \Gamma(\tfrac{5}{2}, \gamma_s \mu_{-1})$$

$$\left. + \frac{\alpha_1^6 - 13\alpha_1^4\alpha_0^2 - 13\alpha_1^2\alpha_0^4 + \alpha_0^6}{384\gamma_s^3 (\alpha_1^2 - \alpha_0^2)^3} \Gamma(\tfrac{7}{2}, \gamma_s \mu_{-1}) + \ldots \right\}. \quad . \quad (35)$$

This is the general expression for the integrated absorption of non-overlapping lines having the Lorentz shape in an isothermal atmosphere and with a constant mixing ratio. When $\alpha_0 = 0$, (35) reduces to the result originally derived by Strong and Plass (1950).

Two limits of (35) are sufficiently important to warrant brief consideration. For a line of any given strength, when the path length is sufficiently small so that $\alpha_1 = \alpha_0 (1 + \epsilon)$, from either (31) or (35) we obtain

$$A(\alpha_0, \alpha_1) = 2\pi\gamma_s \epsilon \alpha_0 \{1 - \tfrac{1}{2}\epsilon\gamma_s + \tfrac{1}{4}\epsilon^2\gamma_s^2 - \ldots\}. \quad . \quad . \quad (36)$$

This converges rapidly when $\epsilon\gamma_s \ll 1$.

On the other hand for a strong line between two points that are sufficiently far apart, the integrated absorption from either (33) or (35) is found to be

$$A(\alpha_0, \alpha_1) = 2\pi^{\frac{1}{2}}\gamma_s^{\frac{1}{2}} (\alpha_1^2 - \alpha_0^2)^{\frac{1}{2}} \left\{ 1 - \frac{\alpha_1^2 + \alpha_0^2}{8\gamma_s (\alpha_1^2 - \alpha_0^2)} + \frac{\alpha_1^4 - 14\alpha_1^2\alpha_0^2 + \alpha_0^4}{128\gamma_s^2 (\alpha_1^2 - \alpha_0^2)^2} - \ldots \right\}, \quad (37)$$

valid when $2\gamma_s \ln (\alpha_1/\alpha_0) \gg 1$.

5. Integrable cases of non-isothermal atmosphere with variable mixing ratio

Some particular examples are presented in this section to demonstrate the rapid convergence of (21) and the equations derived therefrom. For a given sounding it is often possible to represent the function $G(\alpha)$ between two heights with sufficient accuracy as a power of α, so that

$$G(\alpha) = 2\alpha^m \alpha_s^{1-m}, \quad . \quad . \quad . \quad . \quad (38)$$

where α_s is the half-width at some standard pressure and temperature and m is a number not necessarily an integer. From (12) we find the moments μ_l are

$$\mu_l = 2\alpha_s^{1-m} (2l + m + 1)^{-1} (\alpha_1^{2l+m+1} - \alpha_0^{2l+m+1}). \quad . \quad . \quad (39)$$

The quantities λ_l can be obtained immediately from (22). Thus from (21) the integrated absorption is found to be

$$A(\alpha_0, \alpha_1) = 2\gamma_s^{\frac{1}{2}} \left(\frac{2}{m+1}\right)^{\frac{1}{2}} \left(\frac{\alpha_1^{m+1} - \alpha_0^{m+1}}{\alpha_s^{m-1}}\right)^{\frac{1}{2}} \left\{\Gamma(\tfrac{1}{2}, \gamma_s \mu_{-1}) - \frac{(m+1)^2 (\alpha_1^{m+3} - \alpha_0^{m+3}) \alpha_s^{m-1}}{4(m+3)(\alpha_1^{m+1} - \alpha_0^{m+1})^2 \gamma_s} \Gamma(\tfrac{3}{2}, \gamma_s \mu_{-1}) + \ldots\right\}. \quad (40)$$

The various limits considered in previous sections can be derived readily from (40) or from (25), (31), and (33). We do not consider them further here.

In order to compare the transmission functions for different distributions of the radiating gases, we require that the same mass of radiating gas per unit area be above our standard reference level in each case (i.e., different value of m). This requirement can be most easily evaluated if we can also assume that the fractional concentration is represented within a certain accuracy as a power of α, so that

$$\frac{c(\alpha)}{c(\alpha_s)} \left(\frac{T}{T_s}\right)^{\frac{1}{2}} \left(1 + \tfrac{1}{2} \frac{d(\ln T)}{d(\ln \alpha)}\right) = \left(\frac{\alpha}{\alpha_s}\right)^n. \quad (41)$$

As already discussed in Section 2, the temperature factor in (41) is usually very close to unity, so that we may write to a good approximation that

$$c(\alpha) \simeq (\alpha/\alpha_s)^n c(\alpha_s). \quad (42)$$

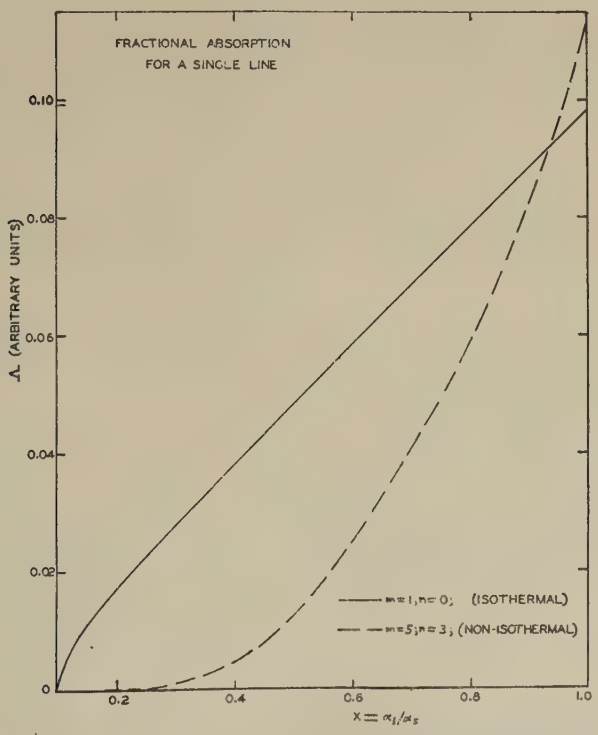

Figure 1. The integrated absorption, $A(0{\cdot}1\,\alpha_s, \alpha_1)$, from a point near the top of the atmosphere where the half-width is $0{\cdot}1\,\alpha_s$ down to the height corresponding to the half-width α_1 as a function of the ratio α_1/α_s, where α_s is the half-width at some standard layer. The solid curve is for a gas with constant mixing ratio in an isothermal atmosphere. The dashed curve corresponds to conditions where both the mixing ratio and temperature very with height and approximately represents average conditions for H_2O. The definition of m and n is given by (38), (41), and (42).

For an isothermal atmosphere, (42) follows exactly from (41) and we find that for this case $n = m - 1$. For a given value of m, if $n \neq m - 1$, it follows that there must be a temperature variation with height. The temperature variation has an important effect on $G(\alpha)$ mainly through the dependence of the line intensity on temperature and to a lesser extent from the explicit temperature factors in (41).

The mass of radiating gas per unit area above a certain height, corresponding to the half-width α_s, in the atmosphere is the same for isothermal conditions and constant fractional concentration, c_0, and for non-isothermal conditions and variable concentration, $c(\alpha)$, if

$$c(\alpha_s) = c_0 (n + 1), \qquad . \quad . \quad . \quad . \quad (43)$$

where $c(\alpha_s)$ is the variable concentration at the level α_s. This result is easily derived from (5) and (41). In all the following examples the total concentration of the radiating gas has been adjusted to be the same according to this definition.

In Fig. 1, the integrated absorption, $A(0{\cdot}1\,\alpha_s, \alpha_1)$ from a point near the top of the atmosphere where the half-width is $0{\cdot}1\,\alpha_s$ down to the height corresponding to α_1 is shown as a function of α_1/α_s. The solid curve is drawn for $m = 1$, $n = 0$. This represents a constant fractional concentration with height ($n = 0$) and isothermal conditions ($n = m - 1$), which are a fair approximation for CO_2. The dashed curve is drawn for $m = 5$, $n = 3$. Here the concentration varies with height ($n = 3$) and the temperature does also ($n \neq m - 1$). This is a good approximation to the average behaviour of H_2O. It is interesting to note that the latter case gives a larger value for A only in the region very close to the reference level α_s, while A is considerably smaller in the latter case at higher altitudes. This suggests that, at the frequencies where the H_2O and CO_2 bands overlap, the H_2O can influence and absorb the outgoing CO_2 radiation only in the lowest layer and that in the higher layers the CO_2 controls the radiation flux.

It is interesting to compare the integrated absorption, A, for (i) a gas with constant mixing ratio in an isothermal atmosphere, but with the absorption coefficient varying with height as the pressure changes and (ii) the same mass of gas per unit area all kept at a constant pressure. $A^{(i)}$ for case (i) is given by (35). $A^{(ii)}$ for case (ii) can be obtained from (40) and (43), when $m \to \infty$ and $n = m -$ constant, since this corresponds physically

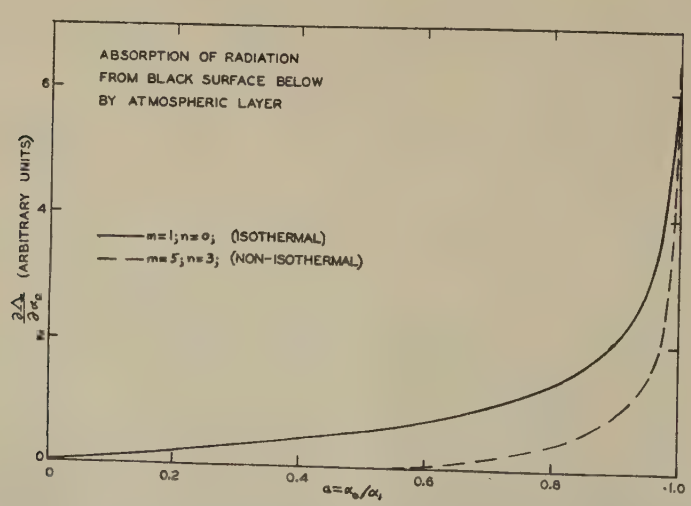

Figure 2. The absorption of radiation from a black surface at a height where the half-width is α_1 by an atmospheric layer of thickness $d\alpha$ at a height where the half-width is α_0 as a function of α_0/α_1. The definition of m and n is given by (38), (41) and (42).

allowing all the gas to be concentrated at the height, α_s. From these equations, it can shown easily that

$$A^{(i)}(0, \alpha_s) = 2^{\frac{1}{2}}(p/p_s)\, A^{(ii)}(0, \alpha_s), \quad . \quad . \quad . \quad (44)$$

here p is the constant pressure used in case (ii) and where it is assumed for simplicity hat $\gamma_s \gg 1$ and that $T = T_s$. This shows for example that the value of A in Fig. 1 $\alpha_1 = \alpha_s$ can never be greater than $2^{\frac{1}{2}}$ times the value for the isothermal, constant mixing-tio case, no matter how much the radiating gas is concentrated near the standard eference height, provided the total mass per unit area is kept constant (the correction or beginning at the level $\alpha = 0{\cdot}1\,\alpha_s$ instead of $\alpha = 0$ is very small). If the pressure hosen in case (ii) is the average pressure along the atmospheric path, (44) shows that $A^{(ii)}(0, \alpha_s) < A^{(i)}(0, \alpha_s)$.

As an example of another type of calculation that can be made from the integrated bsorption, A, Fig. 2 shows the absorption of radiation originating at a black surface at he height corresponding to the half-width α_1, by an atmospheric layer of thickness $d\alpha$ t a height corresponding to the half-width α_0. Since the ordinate is proportional to

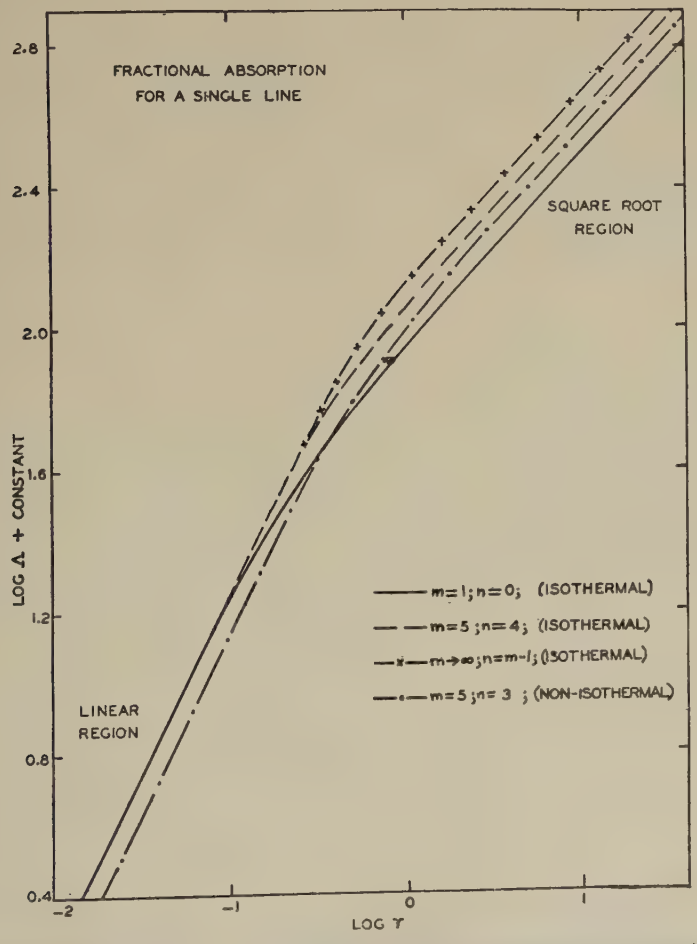

Figure 3. The integrated absorption, $A(0, \alpha_s)$, from the top of the atmosphere to the height α_s as a function of γ_s for four pairs of values of m and n. The linear and square-root regions are indicated. These always occur even for variable path conditions. The definition of m and n is given by (38), (41) and (42). The limit $m \to \infty$ corresponds to a gas entirely at the pressure of the standard level s.

$\partial A(\alpha_0, \alpha_1)/\partial\alpha_0$, it can be easily calculated from (35), (40) and (43). The results have been plotted in Fig. 2 for the cases $m = 1$, $n = 0$ and $m = 5$, $n = 3$ (see discussion of Fig. 1). It is seen that very much more radiation is absorbed in case (i) than in case (ii), at all levels except for those that are very near the surface. Similarly the absorption of radiation by one atmospheric layer at α_1 that is emitted by another atmospheric layer at α_0 can be easily found from these equations, since it is proportional to $\partial^2 A/\partial\alpha_0\,\partial\alpha_1$.

In Fig. 3, the integrated absorption, $A(0, \alpha_1)$, is shown as a function of the dimensionless parameter, γ_s, that occurs in all radiation calculations. It is shown for four different sets of values of m and n, which correspond to various possible variations of pressure and temperature along the path. The linear region and square-root regions discussed after (25) and (33) show very clearly in Fig. 3. For still larger values of γ_s $A(0, \alpha_1)$ increases as a smaller power of γ_s as the lines overlap more and more.

The above example shows that, once the form of the function $G(\alpha)$ has been determined for a particular sounding, (21) and the limiting equations that have been derived from it provide a rapidly convergent series for the integrated absorption between any two heights and thus for the calculation of the radiation flux. This is a particularly simple procedure if $G(\alpha)$ can be represented to the required accuracy by an analytic function which allows the evaluation of the integral in (12) in terms of elementary functions.

For example, in order to represent more complex situations such as temperature inversions, $G(\alpha)$ could be taken to be a sum of any number of different powers of α instead of a single power as in (38). The integral in (12) can still be evaluated readily and the radiation flux obtained from (21). In order to represent the O_3 distribution with its large concentration at great heights, it might be appropriate to represent $G(\alpha)$ by a trigonometric function such as $\alpha \sin m\alpha$. Again the required integrals are readily solved. At a later date we plan to discuss further some of these particular cases and their importance to meteorology.

6. Diffuse radiation

All of the results that have been derived above refer to radiation in a beam making an angle θ with the vertical. All the above equations are functions of θ through γ_s as defined by (7). Although beam radiation is of considerable interest in meteorology, often the final quantity desired is the total radiation integrated over all angles in the hemisphere. A rather remarkable property of radiation transfer between lines that have the Lorentz line shape is that the integration over the hemisphere does not add any essential complexity to the results. As emphasized by Plass (1952b), the actual integration over the hemisphere should be performed where possible, as the technique of multiplying the path length by a constant can lead to erroneous results.

Even for the completely general variation of mixing ratio, pressure and temperature with height that is considered here, the integration over the hemisphere is not difficult. Define γ_s' by

$$\gamma_s = \gamma_s' \sec\theta, \qquad (45)$$

so that γ_s' is independent of θ. In the computation of the total flux of radiation from the hemisphere, the quantity of interest is

$$Y(\alpha_0, \alpha_1) = \pi^{-1} \int_\Omega A(\alpha_0, \alpha_1) \cos\theta\, d\Omega = 2 \int_0^{\frac{\pi}{2}} A(\alpha_0, \alpha_1) \cos\theta \sin\theta\, d\theta. \qquad (46)$$

From (21), it is possible to calculate $Y(\alpha_0, \alpha_1)$ by explicitly writing down the integration over θ and over x (where x is the variable of integration in (19), the definition of $\Gamma(a, b)$) and then reversing the order of integration. The x integration must be divided

to the two ranges $0 \leqslant x \leqslant \gamma_s' \mu_{-1}$ and $\gamma_s' \mu_{-1} \leqslant x < \infty$. In each case the integral ver θ may be evaluated immediately and the remaining integrals are again incomplete amma functions. Thus we find that

$$Y(\alpha_0, \alpha_1) = \sum_{n=0}^{\infty} \frac{\lambda_n}{2^{n-2}\, n\,!\, (n + \frac{3}{2})\, \mu_0^{2n-\frac{1}{2}}} \left[\frac{\Gamma(n+\frac{1}{2}, \gamma_s' \mu_{-1})}{\gamma_s'^{n-\frac{1}{2}}} + \gamma_s'^{2}\, \Delta(-1, \gamma_s' \mu_{-1}) \right], \quad (47)$$

here

$$\Delta(a, b) = \int_b^{\infty} x^{a-1} e^{-x}\, dx. \quad . \quad . \quad . \quad (48)$$

The general expression for the absorption between two arbitrary heights, integrated ver path length, frequency and angle is given by (47) for any variation of mixing ratio, emperature and pressure with height.

The limit of small absorption, even at frequencies near the centre of the line, was onsidered in Section 3 (a). This approximation can never be used for all angles in the emisphere. The mathematical condition for the validity of this approximation, $\gamma_s' \mu_{-1} \sec\theta \ll 1$, can not be satisfied for θ near $\pi/2$. The physical meaning of this is that hat path length between two heights becomes very long as θ approaches $\pi/2$. Thus the bsorption is complete for at least several half-widths from the centre of the line, even or an arbitrarily weak line, near $\theta = \pi/2$. In practice if $\gamma_s' \mu_{-1} \ll 1$, then usually here is only a very small range of angles near $\pi/2$ where the approximations of Section 3 (a) are invalid. These equations may be integrated over the rest of the hemisphere. Otherwise the exact equation (47) must be used in this region.

On the other hand the results of Section 3 (b) can be immediately integrated over he entire hemisphere. The absorption is complete for several half-widths from the line entre when $\gamma_s' \mu_{-1} \sec\theta \gg 1$. Clearly if this condition is satisfied near the vertical, t is satisfied at all angles. Either by integrating (33) over the hemisphere or by taking he appropriate limit of (47), we find that

$$Y(\alpha_0, \alpha_1) = \pi^{\frac{1}{2}}\, \gamma_s'^{\frac{1}{2}}\, \mu_0^{\frac{1}{2}} \sum_{n=0}^{\infty} \frac{(2n)!\, \lambda_n}{2^{3n-3}\, (2n+3)\, (n!)^2\, \mu_0^{2n}\, \gamma_s'^{n}}, \quad . \quad (49)$$

valid when $\gamma_s' \mu_{-1} \gg 1$.

For an isothermal atmosphere with constant mixing ratio, from (34) it follows that

$$Y(\alpha_0, \alpha_1) = \tfrac{8}{3} \pi^{\frac{1}{2}}\, \gamma_s'^{\frac{1}{2}} (\alpha_1^2 - \alpha_0^2)^{\frac{1}{2}} \left\{ 1 - \frac{3(\alpha_1^2 + \alpha_0^2)}{40 \gamma_s' (\alpha_1^2 - \alpha_0^2)} + \frac{3(\alpha_1^4 - 14\, \alpha_1^2 \alpha_0^2 + \alpha_0^4)}{896\, \gamma_s'^2 (\alpha_1^2 - \alpha_0^2)^2} - \ldots \right\}, \quad (50)$$

valid when $2\, \gamma_s' \ln(\alpha_1/\alpha_0) \gg 1$. Similar equations can be written down immediately for he various cases discussed in Section 5.

7. Conclusion

In the absence of appreciable scattering, the radiation flux in the atmosphere can be completely determined from the absorption, $A(\alpha_0, \alpha_1)$, integrated over the path length and over frequencies. A rapidly convergent series (21) has been derived for A which is valid for non-overlapping spectral lines when there is an arbitrary variation of mixing ratio, pressure and temperature with height.

It has been shown that the familiar linear and square-root relationships of infra-red spectroscopy, applicable to constant pressure and temperature conditions along the path, can be generalized to the case when the temperature, pressure and mixing ratio vary arbitrarily along the path. The corresponding relations are that, whenever the atmospheric path is sufficiently short or when the line strength is small (i.e., when $\gamma_s \mu_{-1} \ll 1$),

A varies linearly with γ_s, Eq. (25), and that, whenever the absorption is complete fo several half-widths from the centre of the line (i.e. when $\gamma_s \mu_{-1} \geqslant 1$), then A varies a the square root of γ_s, Eq. (33).

The results for beam radiation at a certain angle to the vertical may be integrate over a hemisphere in order to obtain diffuse radiation. These results for diffuse radiatio are not essentially more complicated than those for beam radiation, as is shown in Section 6

In Section 5 the influence of a mixing ratio varying with height has been considered The results of calculations are given there for the case when this variation together wit the temperature and pressure variations can be fitted with a polynomial. The absorptio of radiation from the ground falls off very much more rapidly with height for a reasonable approximation for H_2O than it does for CO_2, where the total mass per unit area of the two gases is the same and where spectral lines of the same strength are compared.

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Microclimatological profiles in grass cover in relation to biological problems

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(Read 17 July 1954 at the Edinburgh Meeting, Symposium on Meteorology and Scottish Agriculture)

Summary

Selected climatic profiles, as measured by adapted meteorological techniques in the Angus area, were found in general to conform with work done elsewhere either in grass or corn fields. Seasonal and diurnal differences in air-temperature stratification were related to height, density and mode of growth of crop, and solar altitude. The presence of a matted basal layer of dead grass in wild grassland, or the co-existence of a clover crop in hay and silage fields, was noted to be important in maintaining a cool, humid atmosphere next to the ground. Such a zone was favourable to insects when arid conditions obtained at higher levels. In the upper regions where more edible grass leaves extend, wind was shown to be a most significant factor. This was due to the sudden reduction of wind speed within the crop. A steep gradient of wind speed, from zero to 90 or 100 cm/sec, might thus commonly be expected in the region some 40 cm to 15 cm below the upper surface of the stand under a variety of wind and grass conditions. Such a gradient could make available to an insect so situated a considerable range of body temperatures and states of water loss during periods of insolation. Field experiments, however, suggested that convective cooling often extended beyond the depth at which radiation intensity fell off greatly, since, in grass, insect body-temperatures were frequently close to that of the surrounding air. The latter, however, was usually markedly different to that noted in a Stevenson screen. Wind conditions were also noted to be significant biologically wherever experimental procedure required the enclosure of grassland or the proximity of wind-shielding hedges or walls. The ensuing alteration of microclimate particularly as regards air temperature might seriously upset the original purpose of such experiments, a fact noted by workers investigating other crops. Whilst figures quoted might provide a guide to the order of magnitude of some of the microclimatic phenomena encountered in the Angus area due regard should be paid to the lack of uniformity in most grass stands and the sometimes rapid variability of weather conditions.

1. Introduction

The original purpose of the investigation was to examine locally the climate in grass with a view to estimating the conditions under which grass-feeding insects lived. The results obtained were intended to integrate with biological field-observations and also to set up a range of climatic values of use in the design of laboratory experiments concerning body temperature, water loss and locomotory behaviour in intact living insects. These objectives have at the moment only been realised in part. Initial investigations and the examination of the somewhat sparse literature showed that a study of profiles was likely to prove most rewarding especially since movement in a vertical plane was one of the characteristics of the caterpillar-like sawfly larvae under observation. Owing to the general similarity of corn and grass crops, investigations concerning the former by Geiger (1925), Roussakov (1925) and Paton (1948) have proved of inestimable value. Consequently it has been possible, even without continuous recording equipment, to relate isolated observations to a seasonal and diurnal pattern. The profiles to be reported and discussed mainly relate to summer conditions since this is the period when stages of the sawfly caterpillars feeding on grass, clover, and low-growing vegetation are most abundant. Grass cover however can be expected to affect the dormant stages of insects which may be in the underlying soil during the winter, since such cover according to its constitution will undoubtedly influence soil temperature. Further studies would appear desirable ; meanwhile opportunity has been taken to report the findings to date.

2. Methods and apparatus

Thermocouples formed the basis of the apparatus used in noting air temperature, humidity and wind-speed profiles amongst vegetation, and were chosen with a view to adapting the equipment for continuous electrical recording which it is hoped soon to initiate.

(*a*) *Air temperature and relative humidity*

These factors were recorded by instruments similar in principle to Assmann hygrometers with the important difference that by substituting thermojunctions, made of 44 s.w.g. copper/constantan wire, for mercury thermometers the aspirating current was reduced to the order of 5 cm/sec.

(*b*) *Wind speed*

The hot-wire anemometer designed by Yunker (1939) to measure air flow in tubes is suitable for biological experiments for two reasons. First, it is self-regulating for variations in air temperature because it measures the variations in temperature of a current-carrying thermojunction in relation to one at air temperature. Secondly, by taking advantage of the Peltier effect at the heated junction, a heavy heating current is avoided. This is an advantage when the instrument is placed near to living animals. Such an instrument was conveniently spread across an 8-cm diameter perspex frame which was suitable for lowering into grass cover. It was calibrated against a windmill anemometer which was checked by means of Pitot tubes in a small improvised wind-tunnel built to serve this purpose. The instrument was sensitive and easy to read below say 100 to 150 cm/sec but, owing to decreasing deflections with increased air speeds, calibration was not feasible on the galvanometer used much beyond 360 cm/sec. In the conditions here quoted readings outside the crop were not available, being beyond the calibration range.

(*c*) *Insolation*

Insolation was not measured in the field. In the laboratory experiment described in this paper the constancy of insolation over a short period was measured by the voltages generated by differentially connected black and white thermojunctions in vacuo.

(*d*) *Methods*

Various conditions of vegetation varying from pure grass stands to wild conditions with underlying 'mats' of dead grass and intermixed weeds of all descriptions were examined, using the apparatus described. Some of the records concern only a few hours or even minutes and no set of readings relates to an occasion where records were taken over a longer period than 24 hr. The instruments were suspended from a wooden crossbar pierced at either end to slide over two wooden uprights which were driven into the ground in the area selected, with due regard to the currently prevailing wind and incident radiation. Sufficient instruments were used to be able to record at two or three levels. Under steady climatic conditions double this number of levels could be achieved fairly quickly by raising or lowering the crossbar. In the field, instruments were interchanged frequently and routine laboratory checks were carried out. A mercury switch was employed to facilitate rapid successional reading, using one galvanometer.

3. The vertical distribution of air temperature within grass stands

The stratification of air temperature amongst grasses is related mainly to the presence and absence of insolation. Ångstrom (1925) measured the strength of incident radiation at various heights amongst meadow grasses. In the case examined he found that whilst sunshine penetrated considerably in the upper region there was considerable fall off thereafter and practically no direct insolation at lower levels. It is to be expected that the depth of penetration of insolation within the crop will depend upon the height, density and mode of growth of the vegetation concerned and the sun's altitude – factors which vary seasonally and diurnally or both. Since the air in the crop is mainly heated conductively by encountering the warmed grass stalks and leaves, the hottest air level will be found where the crop is beginning to increase in density and the strength of incident radiation has not yet fallen off greatly. Such a level will in general be below the upper surface where the number of grass leaves per unit area is fewest and above the region about halfway down where stalks are more numerous but where incident radiation is reducing rapidly. When a high sun is shining therefore, air stratification in grasses may consist of a warm middle to upper region sandwiched between two cooler layers. The full curve in Fig. 1 illustrates this state of affairs whilst the broken curve illustrates concurrent conditions over adjacent close turf. The dotted curve portrays the conditions in long grass later in the day when a low sun warms only the upper surface of the crop. At night, if conditions are propitious for long-wave radiation, inversions can be expected since the hot upper region will radiate strongly, consequently becoming cool, whilst the cold air will fail to sink due to the close proximity of the grass stalks at lower levels. Fig. 2 illustrates this point.

It remains to consider the effect of seasonal changes in the factors previously mentioned. As growth proceeds from the turf-like condition of the grass B in Fig. 1 to the mature three-year-old grass labelled A in the same figure, the establishment of a middle hot-air layer can be demonstrated as the height of the crop increases. In grass 20 cm high the zones are well established and according to Champness (1950) even in seedling grass. When growth is not so dense the hot-air layer is of course nearer to the

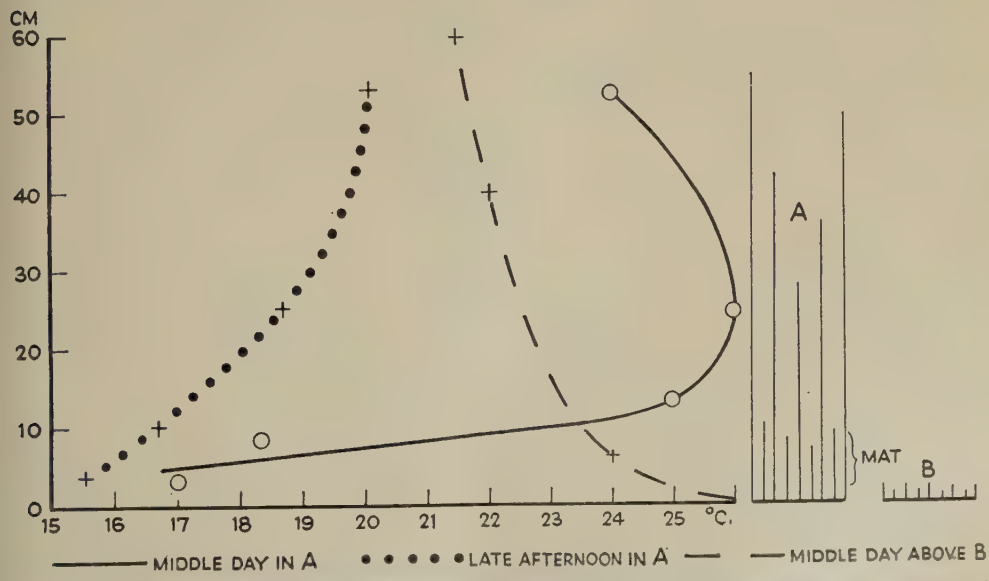

Figure 1. Air temperatures on and above grass on a sunny day. Vertical lines represent grass noted to increase in density towards ground level in a ' mat ' of new and old grass stalks.

ground. On the other hand in autumn, even at mid-day, a low sun radiates only a short vertical distance into the grass and the hot-air layer is consequently at the upper crop surface as indicated in Fig. 3. A similar result may occur earlier in the year if rain or strong winds flatten the grass, when, as shown in Fig. 4, the upper regions become more dense and impenetrable, leading once more to a high hot-air zone.

Since, under normal conditions of growth, the vegetation stand termed grass is usually adulterated with clover in hay and silage crops, or other weeds in wild grassland, the general effect of such intrusions must be considered in all practical cases. It is here that considerable divergence from cornfield conditions is to be noted. Clover undergrowth (Figs. 5 and 8) provides a very dense canopy and thus marks the upper limit of a very cool zone even under summer conditions. So also does the matted basal layer of dead grass and weeds found in uncultivated grassland, already noted in Fig. 1.

The underlying substratum also has a pronounced effect on the climate in the overlying vegetation and much depends upon the thermal conductivity of the soil as indicated by Brunt (1945 and 1946) and Wedmore (1941). In this connection no field work has been attempted in the Angus area although an analogous set of conditions was simulated when making a 24-hour examination of the air stratification in two sets of grass, one growing freely and the other removed with some soil to a shallow box.

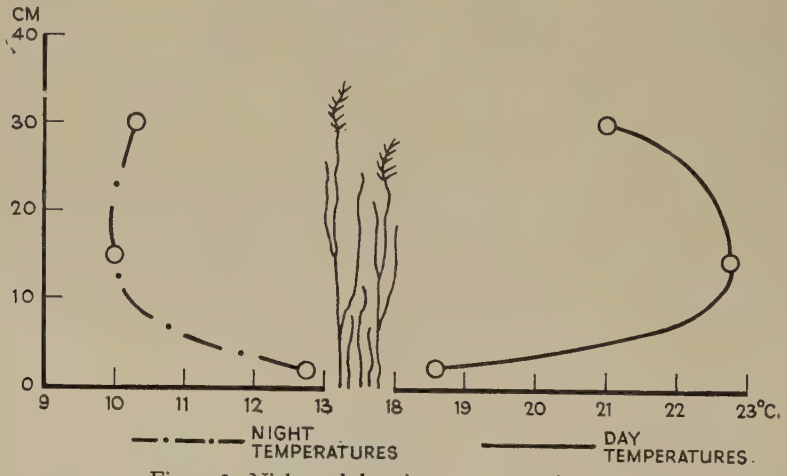

Figure 2. Night and day air temperatures in grass.

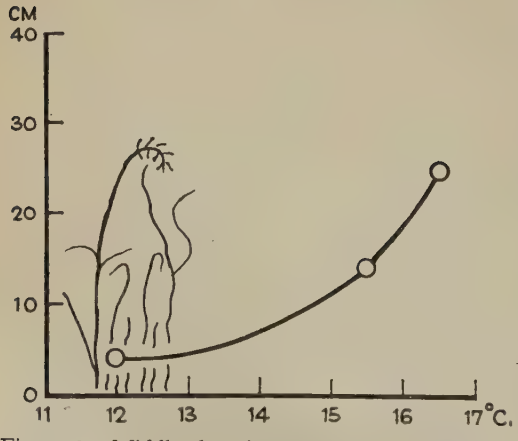

Figure 3. Middle day air temperatures in dying grass in autumn.

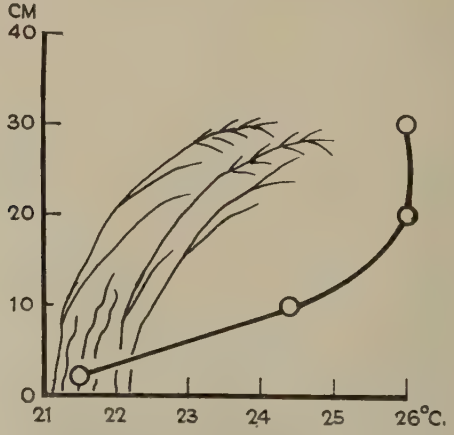

Figure 4. Air temperatures in grass previously flattened by heavy rain.

During a sunny afternoon period the low-level temperatures in the boxed grass were some 2°C above concurrently observed Stevenson-screen temperatures whilst, at a similar station in the free grass, temperatures were low by about the same amount. The difference was presumably due to the insulating properties of the box since upper-region climates in the grass were very comparable. When, however, a muslin screen was used to isolate an area of freely growing grass, although the low-level air temperatures were not greatly altered, those noted in the upper regions were considerably increased due to the prevention of wind cooling. These two experiments indicate possible micro-climatic effects when agricultural and horticultural investigations necessitate such treatments. As found by Daubenmire (1940) in relation to other crops, such experimental conditions may so profoundly alter the microclimate that it may become difficult to assess the success of the original purpose of the experiment. Under this heading exclusive experiments, where screens may be designed to prevent animal or insect access, are particularly suspect. At the same time these Angus experiments indicate what may be expected in the field where substrata vary geologically and hedgerows and walls function as screens.

Summarizing the vertical distribution of air temperature in grass, it can be said that the zonation is approximately that found by Geiger (1925) in cornfields with the proviso that, owing to the greater density of grass vegetation, comparable phenomena are to be noted when crop heights in grass are not nearly so great as in corn.

4. Humidity profiles in grass stands

In all but the shortest grass cover the atmosphere becomes increasingly humid as the ground level is approached. Under the temperature conditions noted in the full-line curve of Fig. 1, whilst the relative humidity in the upper regions of the cover was around 50 per cent, at lower levels in the ' mat ' of dead grass and stalks it remained above 90 per

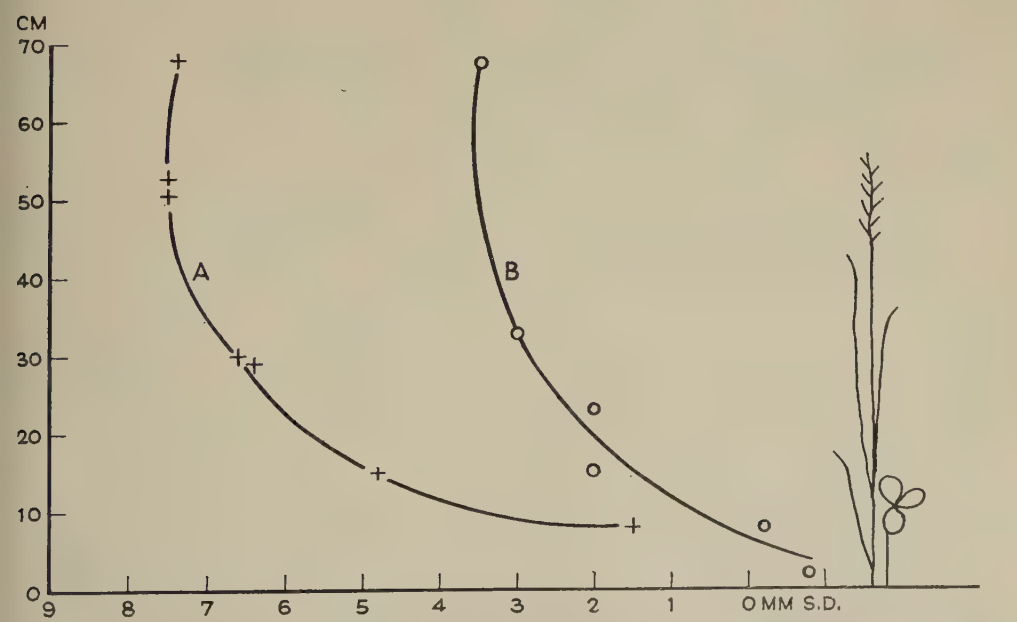

Figure 5. Comparison of saturation-deficiency profiles in the same crop on cold and warm days in summer. Clover undergrowth increases crop density at ground level. Highest grass temperature for curve A, 21°C, and curve B, 14°C (saturation deficiency in mm Hg).

cent. At this time adjacent short turf was completely dried out and withered. The matted layer is usual in wild grassland and, as Milne (1945) points out, may provide a refuge for sheep ticks when conditions higher up would prove unduly desiccating.

Fig. 5 shows a comparison of saturation-deficiency profiles recorded in the same crop on cold and warm days respectively in the month of June. It will be noted that the lower regions, where clover plants replace the ' mat ' illustrated in Fig. 1, remain very moist. Stocker (1923) found high humidities amongst meadow grasses even when very high air temperatures prevailed. Such high humidities are accounted for by the fact that not only does plant cover retard evaporation but it also contributes water from the soil by the process of transpiration. This contribution, whilst rendering the microclimate amongst plant cover even more favourable from a biological point of view, may prove detrimental to certain agricultural practices. Thus Stoddart (1946) found that a Rye nurse crop ' stole ' water from seedlings under protection although presumably conferring the desired humid microclimate.

5. Wind profiles in grass

Wind cools and dries out the air in the upper regions of plant cover but, since it has the same effect directly on insects, the body temperature and rate of loss of internal body fluids of these animals may be considerably disturbed particularly during periods of insolation. For such reasons the study of wind profiles is particularly interesting and it is perhaps remarkable that so little has been done in this connection.

Fig. 6 illustrates a short-period observation made in a silage field on a fairly windy summer day. It will be noted that the wind penetrated about halfway down into the crop with rapidly decreasing speed and degree of fluctuation.

Fig. 7 compares mean wind speeds in different types of grass on different occasions. Some 30-40 cm below the upper surfaces of the crops, wind speed in each case is noted to be practically zero irrespective of the speed at the surface and the constitution of the plants. This was to be expected from a consideration of ' roughness heights ' as made by Paeschke (1937), quoted by Geiger (1951) when the theoretical ground level (where wind speed is zero) varied in grass crops according to height of growth.

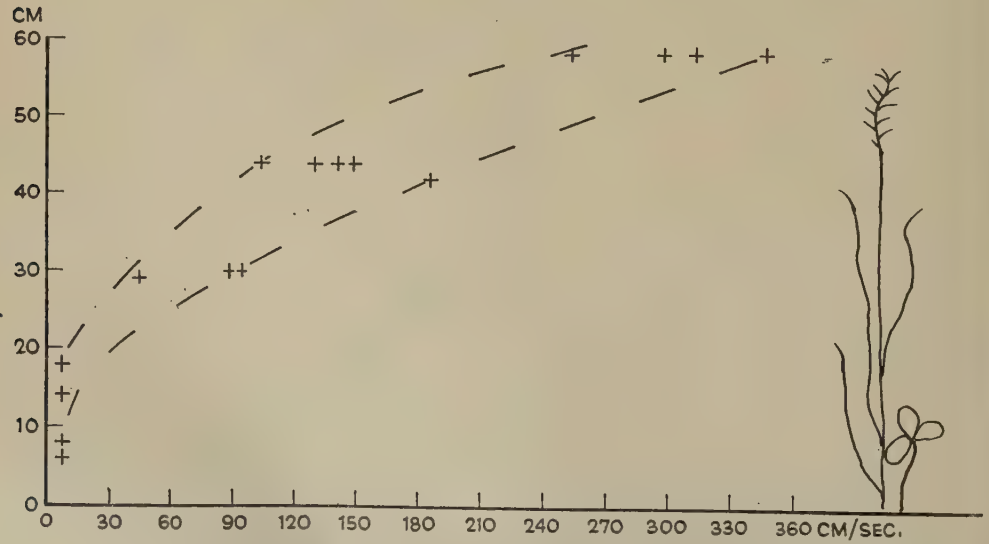

Figure 6. Wind profile in a silage crop. Broken lines denote limits of fluctuations.

Since wind is a common phenomenon in the Angus area it is desirable to make the practical point that, for various macroclimatic wind speeds and various crop heights and conditions, the wind, in the region 15 to 40 cm below the upper crop surface, will frequently be between 100 cm/sec and zero. The significance of these figures will be apparent later in this text when insect body-temperature during insolation periods is considered in relation to wind speeds.

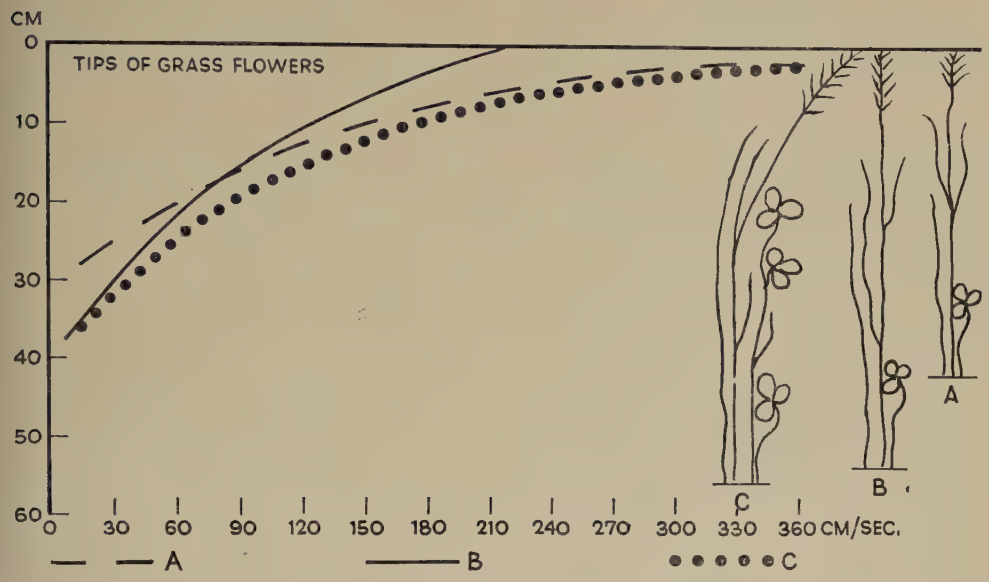

Figure 7. A comparison of mean wind-speed profiles in different conditions of crops under differing microclimatic conditions. Curves are averages of means taken over one hour.

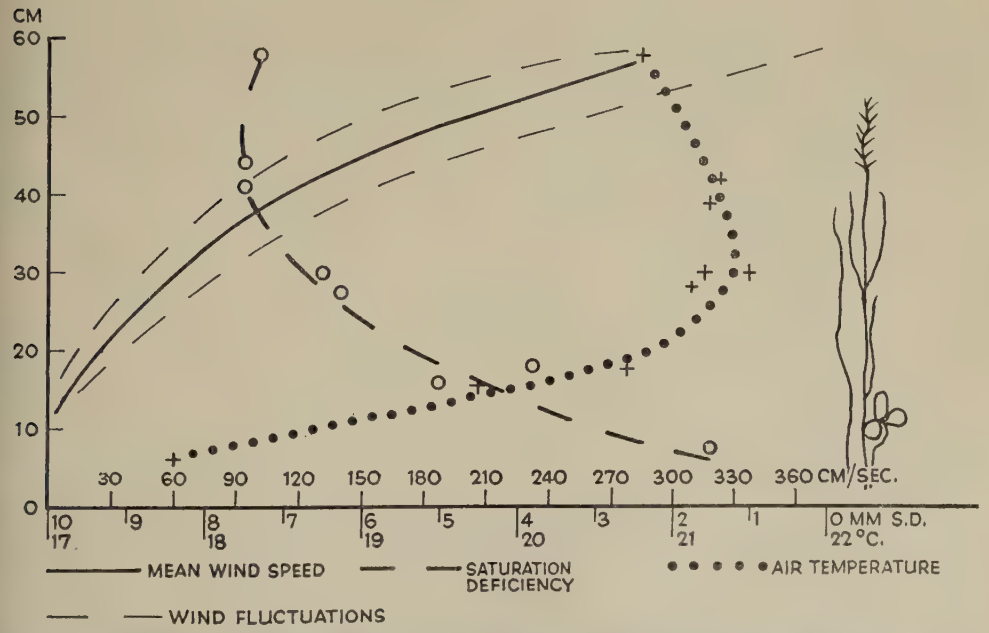

Figure 8. Temperature, humidity and wind profiles in grass between 15 hr and 16 hr GMT, in June under conditions of sunshine after rain on previous day (saturation deficiency in mm Hg).

6. Consideration of the impact of a selected example of grass microclimate on insects.

Fig. 8 gives wind, air-temperature and humidity profiles taken concurrently over a short period in grass 50-60 cm high with clover growing up to 10 cm. The sun was shining and rain had fallen the previous day. For most grass-feeding caterpillars the grass can be divided vertically into a non-feeding zone about the base of the plant and a feeding zone about the middle region where grass leaves, though not etiolated owing to reduced illumination, do not all extend to the upper crop surface where the flowering stalks terminate. Reference to Fig. 8 will show the non-feeding zone to be cool and humid with little or no wind movement. Such a climate whilst possibly not promoting rapid growth, owing to the moderate temperatures there, is conducive to survival, as loss of body fluids can be expected to be a minimum. The region, some 20 to 40 cm above soil level, which may be termed the feeding zone in this case, possesses totally different possibilities for insects. This is largely because both sun and wind penetrate into this region, although air temperature and saturation deficiency are also somewhat greater than at lower levels. Direct insolation may send the temperature of a caterpillar soaring up to 30°C or more, promoting rapid metabolism with the attendant dangers of heat death, or death due to desiccation. The presence of wind, however, will modify insect body-temperature by convective cooling and, simultaneously, reduce the saturation-deficiency gradient at its spiracular openings, with the possibility of a smaller body-fluid loss than might at first thought have been expected had one only considered the increased evaporation due to the air movement. The relationships between insect body-temperature and wind speed for conditions of constant insolation are illustrated in Fig. 9. This graph was compiled from data obtained in a laboratory experiment where the insect (not, unfortunately, a grass feeder) was contained in an improvised perspex wind tunnel and there exposed to natural insolation, constant over the period of the experiment. It will be noted that very slight air movement is sufficient to cool small caterpillars almost to air temperature. In the grass feeding zone remarked in Fig. 8, however, such a wind (*c.* 70-80 cm/sec), only occurs in the upper extremities of the region. Thus, in lower portions of the feeding zone, convective cooling will not be so great and insect body-temperature may be high. Since, however, radiation intensity may be expected to fall off at this point, excessive insect body-temperatures are not so likely to develop in insects feeding in long grass in

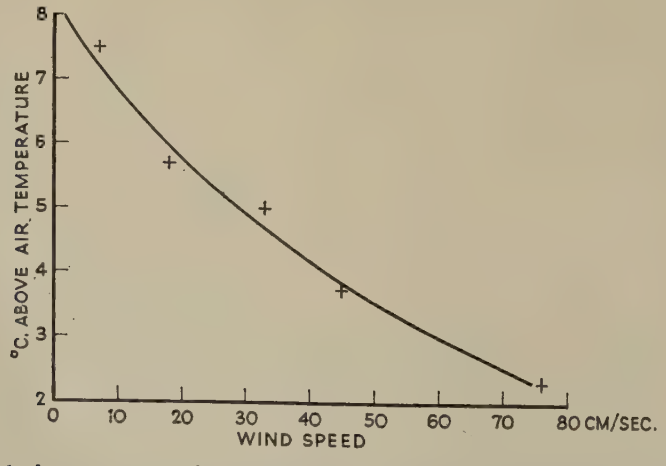

Figure 9. Internal body-temperature of caterpillar of Magpie Moth *Abraxas grossulariata* of weight 0.1140 g with respect to air temperature under conditions of constant insolation and varying wind speeds. Laboratory experiment with air 21°-22°C at 65 per cent R.H.

is geographical area. In fact direct experiment indicates that grass-feeding sawfly terpillars containing small thermojunctions, when lowered into grass stands comparable that in question, do not develop temperatures greatly in excess of that of the adjacent . As indicated earlier, however, the penetration of radiation amongst grasses depends vegetation density as well as solar altitude and in certain stands inimical conditions ight well be expected. In such cases it is worthy of note that the steep vertical climatic adients which may exist amongst grasses are negotiable by locomotory insects so that movement of only a few inches may permit an insect to exchange unsuitable conditions r others optimum to its requirements.

7. Concluding remarks

Owing to the method of treatment it has been more convenient to make such ferences as were desirable in the relevant sections. Much of the information may have nly specific value for the purpose for which it was originally intended, viz., to further ie study of certain grass-feeding insects. However it is hoped that some of the examples nosen may have other uses. The figures quoted should serve as a rough guide to the agnitude of the microclimatic phenomena reported, at any rate for the Angus geographical ea. It is, however, desirable to remember the lack of uniformity in most grass stands. he presence of a few large dock leaves is sufficient to alter considerably the climate a small area, at least as far as an insect is concerned. Finally, in offering examples what are thought to be typical or significant conditions, the discussion of anomalies as been avoided and no stress has been laid on the rapidity with which microclimatic onditions can change. It is therefore desirable now to make this point, since, just as n insect by moving a few inches may vastly improve its conditions, so, sometimes, can favourable microclimate collapse before the animal has time to move.

8. Acknowledgments

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Soil temperatures in an afforested area in Aberdeenshire

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Summary

Continuous records have been maintained since January 1952 of soil temperatures in a forestry plantation near Ballater, Aberdeenshire. The data have led to the following conclusions :

1. The mean annual temperatures are practically the same throughout the soil profile.

2. Inversions of the temperature profile occur in April and August.

3. The frost-free periods have been June to September in the air and May to October in the soil at a depth of 1 in. Frosting at a depth of 6 in. has been restricted to the months of December and January, and no frost temperatures have been recorded at depths of 12 in. or more.

4. During the two years 1952 and 1953 temperatures of 0°C or less have been noted on 167 days in the air, 109 days at a depth of 1 in., and 12 days at a depth of 6 in.

5. There is severe damping of temperature amplitudes with increasing depth, the diurnal variations being very small at depths exceeding 12 in. Estimates of thermal diffusivities have been made for the soil strata from ground level to 1 in. and from 1 in. to 6 in.

6. It is found that good estimates of weekly mean temperatures can be obtained from readings of the daily maximum and minimum temperatures.

1. Introduction

Although it is generally accepted that a forest cover acts as a moderating influence upon temperature fluctuations and consequently on frost occurrence, comparatively few quantitative data are available. Some references are given by Salisbury (1939), in a paper in which he emphasizes the ecological importance of extreme values of temperature as compared with mean values, and Day (1939) examines the mutual inter-relations between forest species and air temperatures at Bin Moss, Aberdeenshire. Soil temperatures have been observed in Idaho, California and Ohio by Daubenmire (1943), Fowells (1948) and Dreibelbis (1951) respectively, and in the Moscow region by Vasilev (1952). Orlov (1954) found that, in Siberia, larch roots are concentrated in layers where the maximum temperature exceeds 4°C to 5°C. Mork (1933) made an extensive study of conditions in Norway, comparing temperatures in the air and at different depths in the soil, both in forests and in clearances. In view of the increasing importance of forestry in Scotland it was thought desirable to obtain data applicable to conditions in this country, and this paper gives an account of some of the results obtained during the first two years of the investigation.

2. Site and instruments

The site selected is on a hillside in the Alltcailleach Forest, Birkhall, at an altitude of 1,100 ft, where conifers were planted in 1937, and are now 8 to 10 ft high. The soil is moderately podsolised and the profile exposed in a demonstration pit a few yards from the soil thermometers is described as follows (FitzPatrick 1951) :

ocality : Corrie Burn basin. Ride between compartments 41 and 42, 20 yd from the Corrie Burn.

ope : Micro 3° 11′; macro 8° 35′. Top of knoll with easterly aspect.

rainage : Profile drainage free and surface run-off definite.

rent material : Glacial drift.

escription : 0-1 in. (A_0) – Sharply defined, smooth, black, sandy, coarse mor-mull type, speckled with bleached sand grains, stoneless, friable with loose crumb structure and 4 per cent small stones and gravel. Numerous young and old healthy roots.
1 in.-6 in. (A_1) – Clearly defined, coarse sandy loam, speckled with bleached sand grains. 5 per cent very small sub-angular stones and gravel. Loose and non-porous in appearance, friable crumb structure when wet. Numerous fine healthy roots, earthworms occasional.
6 in.-12 in. (B_1) – Clearly defined, wavy, coarse sandy loam with frequent large stones and boulders. 10 per cent small stones and gravel. Loose, non-porous with friable crumb structure when wet. Occasional fine roots.
12 in.-18 in. (B_2-B_3) – Clearly defined, irregular, coarse sandy loam, with frequent large stones and boulders : 15 per cent small stones and gravel. Structureless, slightly indurated with occasional very fine roots, merging into (B_3) region with abundant large stones and boulders. 30 per cent sub-angular stones and gravel, indurated, roots absent.
18 in. (C) – Boulders and stones dominant, 35 per cent stones and gravel. Cloddy, strongly compacted with small spherical discontinuous pores. Roots absent.

A thermograph, housed in a Stevenson screen at 4 ft above ground level, gives a ontinuous record of air temperatures, and four mercury-in-steel thermometers, in onjunction with two Negretti and Zambra dual-pen recorders, are used for the measure-ent of soil temperatures. The soil thermometers were buried at the edge of a ride rough the plantation, where there is some shading from the trees : the total depth of e A_0 horizon quoted above includes the small amount of litter present. Disturbance the natural soil at the position of the thermometers was avoided by digging a pit with vertical face and fitting the thermometers into horizontal holes bored into this face at pths of 1 in., 6 in., 12 in., and 18 in., which correspond with the boundaries of the tural horizons in the profile. After inserting the thermometers, the pit was refilled. The struments were installed in December 1951 and complete data for the soil temperatures e available for the years 1952 and 1953, but there is a gap in air-temperature records account of instrumental difficulties in the autumn of 1953. The charts on the instru-ents were changed weekly but, as it was not possible to visit the site daily, a raingauge d a grass minimum thermometer were installed near the Forestry Commission's Birkhall wmill, about a mile from the plantation; the rainfall results are shown in Table 1.

TABLE 1. Monthly rainfall (mm) at Birkhall sawmill, 1952/1953
(Figures in brackets show number of days upon which reading included melted snow in the gauge)

	J	F	M	A	M	J	J	A	S	O	N	D	Total
2	59·2	27·7 (9)	56·1 (6)	22·1 (1)	52·8	62·5	44·2	167·1	35·3	134·6	83·8 (3)	87·6 (4)	833·0 (23)
3	24·1 (1)	39·9 (7)	7·9	71·6 (2)	44·2	85·1	142·0	40·6	76·7	37·3	77·0	86·6	733·0 (10)

Under these conditions of rainfall, the soil retains a considerable amount of moisture roughout the year, and the influence of the organic surface layer is illustrated in Table 2 hich shows the moisture content of samples taken at the demonstration pit at the end May 1953.

TABLE 2. Soil moisture at demonstration pit, May/June 1953
(Percentages of 2 mm-sieved soil dried at 105°C)

Depth (in.)	0–1	1–6	6–12	12–18
Water (per cent)	93	64	17	21

Weekly average temperatures were obtained by integration of the temperatur charts; these averages are referred to as true mean values. Information was also derive for extreme values, amplitudes of fluctuation, freezing periods, and any incidental feature of specialised interest. As an example of one such incident, reference may be made t the effect of a violent rain-storm (15 August 1952) when there was a precipitation o 4 in. in $2\frac{1}{2}$ hr. In consequence of this, there was an abrupt fall in temperature of 2·7°C at the 1 in. depth and of 0·9°C at the 6 in. depth, with barely perceptible changes i the lower horizons. It was not possible to measure moisture conditions in the profil immediately after the storm but the temperature changes appear to be in accordanc with the data of Table 2 in suggesting that the greater part of the water is retained i the upper strata of the soil.

The figures presented in the Tables in this paper show that the year 1953 was both drier and warmer than the year 1952; the rainfall for 1952 was slightly lower than the mean value (850·9 mm) obtained at the nearest long-established raingauge (at Ballater).

3. Temperature data

True mean annual temperatures are shown in Table 3; in order to economize space monthly averages are not shown, but it should be mentioned that the higher values in 1953 are largely accounted for by the unusually mild conditions in the earliest and latest months of the year. Absolute maximum and minimum values are also shown (Table 3).

TABLE 3. Absolute maximum, minimum and annual mean temperature (°C) in Alltcailleach Forest 1952/1953, in air and at various depths in soil

		Air	Soil, depth (in.) 1	6	12	18
1952	Max.	26·6	20·6	15·2	12·7	11·8
	Min.	− 11·1	− 1·7	0·0	0·5	1·0
	Mean	5·9	5·8	5·8	5·9	5·9
1953	Max.	24·3	18·4	14·6	13·7	12·0
	Min.	− 8·3	− 1·3	0·0	0·7	1·6
	Mean	–	6·8	7·0	6·9	7·0

It will be noticed that in each year the mean annual temperatures are practically the same throughout the profile – a general result that applies also to an extensive series of observations under a turfed soil at Oxford (see Gloyne 1950) but not to the case of a bare soil at Rothamsted (Keen and Russell 1921, p. 223). The general form of the annual variations is shown in Fig. 1, where it will be seen that inversions of the temperature profile occur at about the beginning of April and the end of August; similar changes occurred in 1953, but in that year there were subsequent small fluctuations, and the reduction in temperature at 1 in. only became appreciable in October.

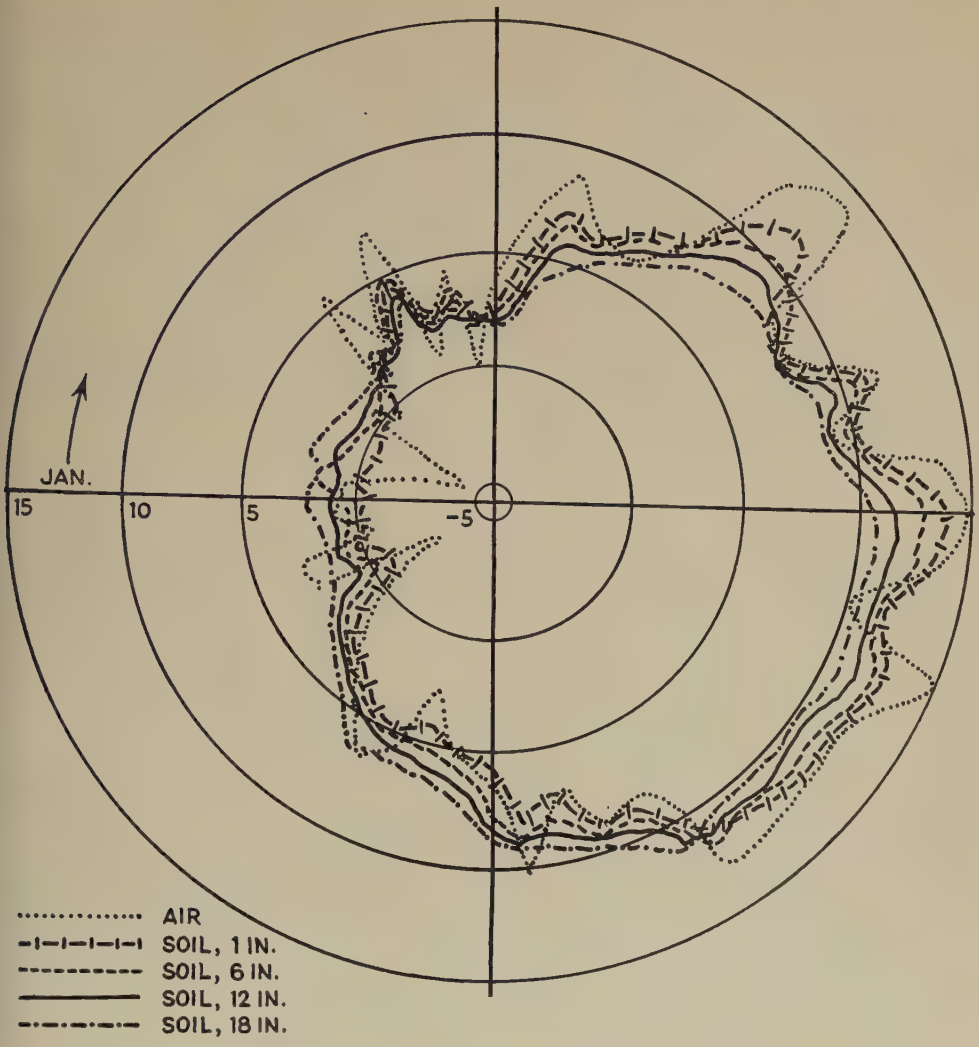

Figure 1. True weekly mean temperatures, 1952.

TABLE 4. NUMBER OF FROST-DAYS IN ALLTCAILLEACH FOREST, 1952/1953, IN AIR, AND AT VARIOUS DEPTHS IN SOIL

		J	F	M	A	M	June–Sept.	O	N	D	Total
952											
	Air	26	18	15	5	1	0	3	14	17	99
	Soil, depth (in.)										
	1	29	21	2	0	0	0	0	7	28	87
	6	5	0	0	0	0	0	0	0	4	9
	12_+	0	0	0	0	0	0	0	0	0	0
953											
	Air	12	12	16	13	3	0	0	2	10	68
	Soil, depth (in.)										
	1	9	7	5	1	0	0	0	0	0	22
	6	3	0	0	0	0	0	0	0	0	3
	12_+	0	0	0	0	0	0	0	0	0	0

4. Frost-days

The weekly true mean temperatures plotted in Fig. 1 show relatively few cases in which the soil temperatures fall to freezing point, but from the ecological point of view it is of interest to consider in rather more detail the frequency and times of occurrence of frost. This is done in Table 4, which shows the essential features disclosed by an examination of the recorder charts.* The dates of the latest recorded spring air frosts were 8 May 1952 and 8 May 1953, and the earliest autumn frosts were 11 October 1952 and 12 Nov. 1953, so that there is a frost-free period of five to six months. Grass minima were, of course, lower than the air temperatures recorded in the screen, and showed extreme values of 11·1°C to − 14·4°C in 1952 and 12·8°C to − 15·6°C in 1953; grass minimum temperatures below 0°C were recorded in every month of 1952 and 1953. At the 1 in. depth, there has been one observation of spring frost as late as April, but none earlier than November in autumn, and, at 6 in., frosting has been restricted to the months of December and January.

5. Diffusivity

The results quoted above for the relatively slight frosting at even small depths below the surface are in accord with data obtained elsewhere. This rapid damping of surface temperature fluctuations corresponds to a low value for the diffusivity of the surface layers and an attempt has been made to estimate this factor numerically. Unfortunately the results for the surface layer are only very rough approximations, for, in addition to the usual assumptions made in the calculation (*vide infra*), no data were available for the surface temperatures, and there are probably considerable errors involved in using the air temperature in the screen as an estimate of the surface temperature. Results such as those obtained by Fowells (1948) and Geiger (1950) show that there are appreciable gradients in air temperature near the surface, especially if the soil is exposed to direct insolation. Under forest conditions, it is probable that the ground temperature is slightly lower in winter (Day 1939) and higher in summer than the screen temperature. It is however worth while calculating the diffusivities in order to compare them as regards order of magnitude with the more accurate values calculated for the layer of soil between 1 in. and 6 in. below the surface. At 12 in. and 18 in. the amplitudes are too small for calculations to be made.

It has been suggested recently (Azuma 1952) that diffusivities can be obtained more readily from observations of time lags of the maxima and minima than from comparisons of amplitudes, but with the present data, the variations in the slopes of the temperature-time charts are too slow to assess with accuracy the times of occurrence of the turning values. The diffusivities have therefore been calculated by a modification of the method used by Keen (1931).

Keen and Russell (1921, p. 229) found that the ratio of the soil amplitude to the air amplitude increased as the air amplitude decreased, but the graphs (Fig. 2), relating the two amplitudes to one another in the present data appear to be essentially linear, with a more or less symmetrical scattering of the points about the straight line derived by the method of least squares. We are therefore justified in calculating in each case the ' best ' straight line (according to the method of least squares) from all available data for the amplitudes concerned, and using the slope of this line as the mean value of (θ_1/θ_2), the ratio of the amplitudes. Theoretically this line should pass through the origin ($\theta_1 = 0$, $\theta_2 = 0$), but, as found by Keen, this was not the case in practice. It was noted, however,

* As the autumn of 1953 was mild, it is unlikely that any frost temperatures in the air have been omitted by the break in the thermograph records in October and November.

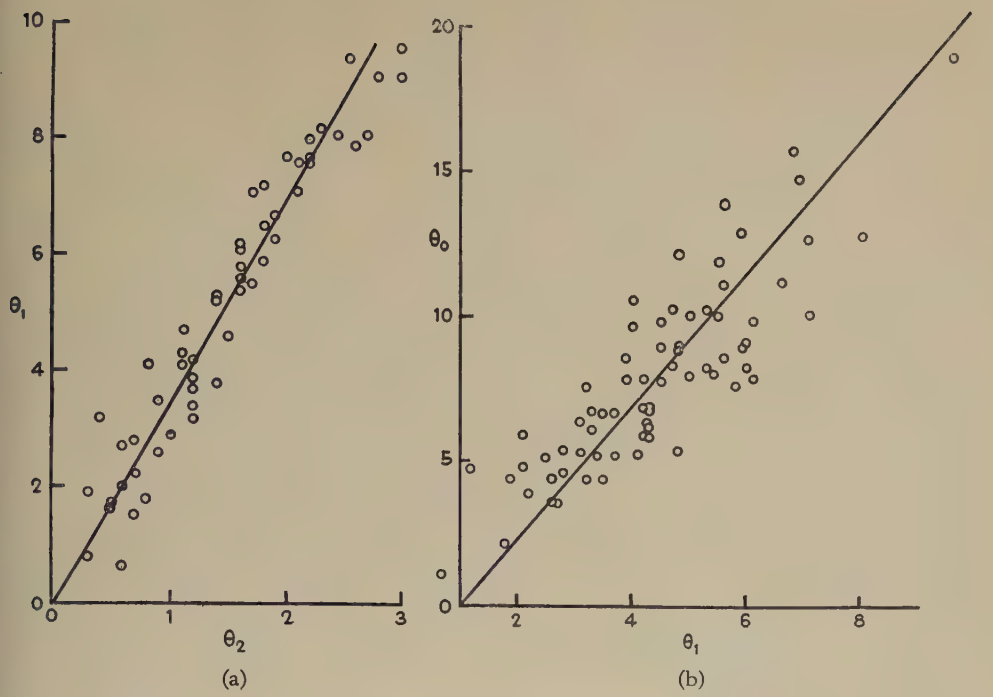

Figure 2. Relationships between diurnal amplitudes of temperature (*a*) at depths of 1 in. and 6 in., θ_1 and θ_2; (*b*) in air and at depth of 1 in., θ_0 and θ_1.

that the deviation was smallest when the number of observations was largest, and its presence was probably due to departure from the simple law of temperature variation implied in Eq. (1), below, rather than to thermometric errors.

It is usually assumed, as an acceptable approximation to actual conditions, that the fluctuation about the mean temperature at time t and depth x below a plane of reference $x = 0$ is given by

$$\theta = \theta_0 \exp\left[-x\left(\frac{\pi}{kT}\right)^{\frac{1}{2}}\right] \sin\frac{2\pi}{T}\left[t - \tfrac{1}{2}x\left(\frac{T}{\pi k}\right)^{\frac{1}{2}}\right] \quad . \quad . \quad (1)$$

where T is the period and k the diffusivity. Then the ratio of diurnal amplitudes θ_1 and θ_2 at depths x_1 and x_2 is given by

$$\frac{\theta_1}{\theta_2} = \exp\left[(x_2 - x_1)\left(\frac{\pi}{kT}\right)^{\frac{1}{2}}\right] \quad . \quad . \quad . \quad (2)$$

and the time of phase lag at the lower stratum, $x = x_2$, relative to the upper stratum $x = x_1$, is given by τ where

$$\tau = \tfrac{1}{2}(x_2 - x_1)\left(\frac{T}{\pi k}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (3)$$

It follows immediately from Eq. (2) that

$$k = \frac{\pi}{T}\left[\frac{x_2 - x_1}{\ln \theta_1/\theta_2}\right]^2 \quad . \quad . \quad . \quad . \quad (4)$$

In the customary CGS units k is expressed in cm^2/sec, and $T = 86{,}400$ sec. The results for k are shown in Table 5. The results for the layer 1 in. to 6 in. are fairly accurate

TABLE 5. VALUES OF DIFFUSIVITY, k, (CM^2/SEC), CALCULATED FROM n PAIRS OF AMPLITUDES

Depths (in.)	1952				1953			
	0–1		1–6		0–1		1–6	
	n	$k \times 10^4$	n	$k \times 10^3$	n	$k \times 10^4$	n	$k \times 10^3$
Jan.–Mar.	23	1·1	14	5·7	33	1·2	29	5·4
Apr.–June	73	3·3	71	2·7	87	5·1	85	3·7
July–Aug.	51	5·2	48	3·5	47	2·3	55	4·1
Sept.–Dec.	56	3·5	37	4·2	15	1·5	65	4·1

(and with Eq. (3) give a time lag of 4 hr to 5 hr, in agreement with the somewhat variable times actually observed). For reasons already mentioned, the numerical values for the surface layers are not reliable, but they are certainly much lower than those in the deeper stratum, and approximate to those usually found for organic soils and peats. With records for two years only, the data are insufficient to attempt an investigation of the quantitative variations due to changes in moisture content, a problem which has attracted attention since the pioneering work of Patten (1909). The values of k calculated in Table 5 have been limited to periods within which moisture changes were on the whole fairly small : no physical significance could be attached to a value derived from data averaged over a whole year.

6. EFFICIENCY OF SPOT READINGS

Various workers have attempted to establish the validity of ' spot readings ' which can be used in the absence of continuous records to give reliable mean values of daily or weekly temperatures. It is well known that the conventional temperature reading at 09 hr does not give a satisfactory estimate of the mean air temperature. This was noted, for example, by Gloyne (1950), and, from the present records, it was found that the true mean weekly air temperatures could be equated to the 09 hr temperatures only when the diurnal variations were small, and that they were consistently over-estimated in summer by between 1°C and 2°C. Even if errors of this order are acceptable for some purposes, the 09 hr readings are obviously inappropriate for estimating mean soil temperatures : in our own case, at a depth of 6 in., the time lag is such that the 09 hr value approximates to the *minimum* soil temperature. Mork (1933) found that the mean soil temperature between 06 hr and 18 hr was given to a close approximation by the average of the temperatures at 08 hr, 14 hr and 19 hr, and West (1951) successfully estimated the diurnal means from readings at 0830 hr, 12 hr and 17 hr, but there must be many routine stations which cannot be visited thrice daily. If there were a simple harmonic variation of temperature with time, the mean of the maximum and minimum temperatures would give the exact value of the mean temperature; although the actual temperature fluctuations differ from this ideal case, it has been found (e.g. Keen and Russell 1921, p. 219) that this procedure gives reasonably satisfactory results when applied to the estimation of mean weekly temperatures. The application of this method to the present data is shown in Table 6, which gives a classification of the deviations (regardless of signs) from the true means of the weekly means estimated from daily maximum and minimum readings. If $\pm 0{\cdot}5$°C be taken as an estimate of the error that can be tolerated in readings at a routine station, it will be seen that for the air temperatures 69 per cent fall within these limits (and 90 per cent within the limits of $\pm$ 1°C); for the soil temperatures at depths of 1 in. and 6 in. the percentages with the limits $\pm$ 0·5°C are 89 per cent and 100 per cent respectively.

TABLE 6. Deviations from true weekly means of means estimated from daily max. and min.

	Deviations (°C)									
	0–0·5	0·5–1	1–1·5	1·5–2	2 +		0–0·2	0·2–0·3	0·3–0·5	0·5–1	1 +
						Soil depth 1 in.	49	20	15	9	1
	62	19	5	2	2	6 in.	77	9	7	0	0

Since the observations of daily maximum and minimum temperatures require only ıe inspection per day of the instruments, this method appears to be a useful one if ɔntinuous recordings are not available.

Acknowledgments

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Climate, host and parasite in crop disease

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(Read 17 July 1954 at the Edinburgh Meeting, Symposium on Meteorology and Scottish Agriculture)

Summary

The effects of climate on both host and parasite have been studied for two serious crop diseases of west Scotland.

First-early Epicure potato crops in the coastal belt of Ayrshire and Wigtownshire make appreciable tuber growth before the soil temperature rises to 43°F, which is the minimum temperature of activity of the potato-root eelworm, *Heterodera rostochiensis*. By the time the earliest crop is lifted early in June, cysts of the eelworm are white or yellow in colour. It is shown that such cysts make little contribution to the permanent soil infestation, thus explaining the low infestations of land consistently devoted to the growth of first-early crops. Later lifting allows the cysts to become brown, and such cysts add considerably to the permanent soil infestation. The initial period of eelworm-free growth on first-earlies limits the loss in yield to 30 to 40 per cent, as compared with 80 to 100 per cent loss if the variety Epicure were planted later, at the time for a maincrop variety.

Correlation coefficients have been calculated for comparisons over eight years between the percentage of club-root disease on swede turnips and monthly means of air and soil temperatures, bright sunshine and rainfall, for the separate months from May to October. Significant and near-significant correlations show that a low incidence of the disease is associated with a relatively high soil temperature in July, relatively high amounts of bright sunshine in June, July and September, relatively low rainfall in September and relatively high rainfall in October.

The differential effect of soil temperature on host and parasite with potato-root eelworm is a 'disease-escape' mechanism but the effects of bright sunshine on the amount of club-root disease are considered to be, in the present setting, an effect upon disease involving growth of the host.

1. Introduction

There is a growing tendency in plant pathology to examine in greater detail the part played by the host plant in the dual relation involved when a parasitic organism attacks it. The effects of weather on crop disease have, hitherto, reflected mainly an influence on the parasites. This influence is nevertheless very important in practice. It has, for instance, been found possible (Beaumont 1947) to forecast the first appearance of potato blight, following the occurrence of suitable climatic conditions which apparently favour the fungus *Phytophthora infestans*. Important practical forecasting can indeed be accomplished on a basis of this knowledge (e.g. Large 1953, Grainger 1950, 1953). The writer has found, and used in practice, relations between the number of hours per week of 100 per cent relative humidity and the severity of corn mildew, *Erysiphe graminis* (Grainger 1947). Sirry (1953), working with the writer, also confirmed a relationship between the average relative humidity and the attack of the fungus *Botrytis cinerea* on lettuce. In the last two diseases the effect of average relative humidity is an effect on the fungus, probably reflecting the frequency with which 100 per cent R.H. is attained, for spores of both species will germinate at 100 per cent R.H., but not at lower values. Numerous other instances of the effect of climate on disease, usually affecting the parasite more than the host, are given in two detailed reviews by Foister (1935 and 1946).

The discovery of 'masking' of virus disease symptoms introduced an effect of temperature acting on both host and parasite. Johnson (1921) reported that symptoms of mosaic virus disease were masked when an infected tobacco host was grown above 98°F. New leaves made under these conditions appeared healthy, though the presence of virus could be demonstrated by re-inoculation from such leaves to healthy plants grown at normal temperatures. The writer (Grainger 1936) found a similar masking below 50°F, and later (1939) showed that there were important differences between the

ctivity of the virus and its host between 70°F and 85°F. The virus has a narrower and ifferent range of activity in relation to temperature from that of the host, and masking of ymptoms above 98°F and below 50°F are but manifestations of low virus activity at the xtremities of the temperature range of the host plant.

Differential relations of a fungal parasite and its host to temperature are also known. Good examples are reviewed in Butler and Jones (1949). Temperature is, however, usually ill considered for its effects on the parasite, and its action on the host, in so far as it ffects the amount of disease and not merely the growth of a healthy plant, has not been tudied extensively. A preliminary indication that such host effects could be important as obtained when the writer was extending the work of Sirry (1953). The fungus *Botrytis cinerea* has an optimum temperature of growth in culture round 70°F. The ame optimum temperature appeared to apply to development of disease on the lettuce rop in March, but in June the optimum value was appreciably lower. This difference ould seem to involve growth of the host in addition to activity of the parasite. It may e an effect of temperature acting through other factors such as length of day or light ntensity, but is still an effect which involves growth of the host.

The purpose of this paper is to explore the relative effect of climatic factors on both ost and parasite in two diseases severe in the west of Scotland, namely potato-root eelworm nd club-root disease of turnips and swedes.

2. Potato-root eelworm and the early crops

A narrow strip of land along the coast of Ayrshire and Wigtownshire is used for the rowth of early Epicure potatoes. The earliest crops, planted in February and lifted in une, are grown very near to the sea-shore, in relatively frost-free situations. They are rown year after year without a break but, in spite of this lack of rotation, soil infestation ith the potato-root eelworm, *Heterodera rostochiensis*, is quite low. Paying crops are, oreover, obtained every year, although most of the land is eelworm-infested and a loss f yield amounting to 30 to 40 per cent is experienced. This contrasts with a loss of 0 to 100 per cent on maincrop potatoes grown in other areas, where eelworm infestation s spreading slowly, but inexorably. It is important to find the reasons for the relatively light effect of the eelworm on the first-early crop and for the low soil infestations which ccur. Such a type of culture indeed represents what is apparently the only alleviation f this serious disease by methods available to the farmer.

There are, in the minds of practical growers, several suppositions to account for this elatively favourable setting of the first-early crops. It is assumed that the low soil emperature in the early months of the year restricts the activities of the eelworm but ot of the potato, thus giving a period of eelworm-free growth. This accounts for the elatively small loss in yield. Restriction of eelworm activity is such that it is still immature t the time of early lifting. This would render it incapable of contributing largely to oil infestation. It therefore seemed necessary to compare soil temperature with the ourse of tuber production, and also to investigate the ability of the eelworm to increase he soil infestation at the time the early crop is lifted.

The potato-root eelworm produces minute cysts – fertilised female eelworms which ease to exist as individuals, fall from the potato roots on which they are formed, and ecome virtually bags of eggs lying free in the soil. These eggs hatch into larval eelworms, articularly under the influence of secretions from potato roots, but only when the soil emperature rises above the minimum value of 43°F (O'Brien and Prentice 1932). This alue is considerably lower than that given by Chitwood and Buhrer (1946) for the J.S.A. They obtained evidence of eelworm dormancy at weekly mean temperatures

up to 54°F. It is clear that the potato-root eelworm is active at temperatures very much below 54°F in Ayrshire; it has, indeed, often completed its life cycle before the soil temperature attains that value. O'Brien and Prentice have laid down, for Ayrshire, an approximate time schedule for various stages in the life history of the eelworm. It required about 14 days for the larvae to move from cysts in the soil to the roots, and new cysts could first be recognized on the potato roots some 48-52 days after the larvae became active. They are at first white, until after about 70 days; between about the 70th and the 80th days they are yellow, and become brown after 80 days. Although the female eelworm is fertilized and, anatomically, sexually mature when it can be seen on the roots it has been assumed that the white and yellow cysts are not as fully mature as the brown.

In order to test this assumption, roots with white, yellow and brown cysts were taken from plants as they were dug, the whole mass of roots was weighed, and small portions cut in random manner from each. These random samples were weighed, and the cysts upon them counted. A weight of the original sample was then calculated so that when it was added to 10 kg of eelworm-free soil it gave an infestation of approximately 0·4 cyst per gram of soil. Three pots each containing 10 kg of soil were so treated, and a further three pots were given a double infestation of about 0·8 cyst per gram. A washed seed potato was set in each pot and allowed to grow. When growth ceased, the soil from each pot was mixed thoroughly and again sampled for the estimation of the new eelworm cyst population. Potatoes grown in the original eelworm-free soil remained free from eelworm, thus providing a control. The results are given in Fig. 1, and it will be seen that the cysts originally white or yellow had only increased by relatively small numbers, but the cysts originally brown had increased to at least five times the numbers from either white or yellow. This result therefore shows that white and yellow cysts are indeed far less able than brown ones to add to the soil infestation. It still remains to consider the effect of the soil temperature on tuber production and also as it affects the stage of cyst development at lifting.

In the year 1949, potato plants were lifted at intervals from eelworm-infested fields near Girvan, Ayrshire, to estimate the amount of tuber production through the growth

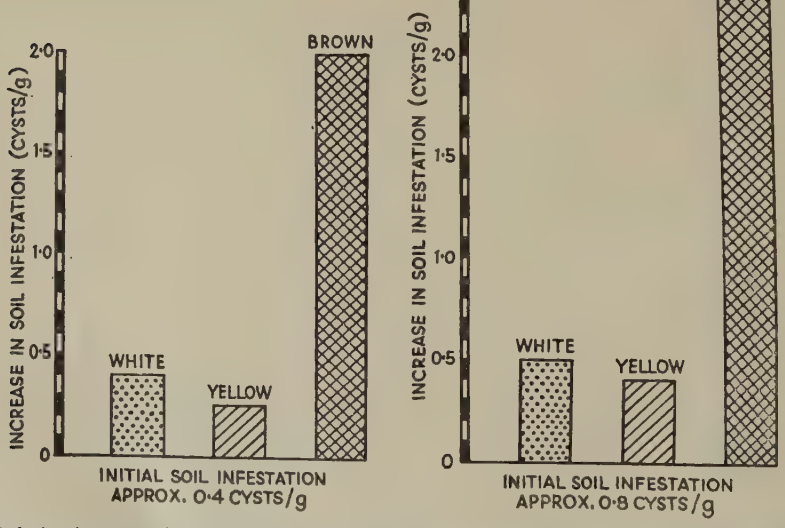


Figure 1. Relative increases in cyst population following the growth of potatoes in soil containing approximately equal initial numbers of white, yellow and brown cysts, added at two initial levels.

cycle. The meteorological station at Auchincruive is within the coastal strip on which Epicure potatoes are grown, though not in the first-early belt. It is, however, the only station within the strip which publishes records of soil temperature. There are four 'maritime' stations reporting soil temperatures in the meteorological district of Scotland W., namely Dunoon (Benmore), Rothesay, Auchincruive and Dumfries. There is, in most years, a very close agreement in the soil temperature readings of these stations during March and April, when values round 43°F are to be expected. In 1949, the maximum difference between them was 0·7°F in March and 0·4°F in April. The Auchincruive figures varied by only 0·1°F from the mean value of the four stations (*Monthly Weather Report* 1949) and would therefore seem typical of a wide range of coastal situations, and suitable for comparison with the periodical liftings in the Girvan area.

Most of the cysts of potato-root eelworm are found in the top 9 in. of soil, but there are usually sufficient at greater depths to make the soil temperature at 1 ft a relevant value.

Fig. 2 compares the tuber-production cycle with soil temperature. The value of 43°F was, in 1949, attained in the last week of March and the larvae would, according to the time-table of O'Brien and Prentice (1932), have moved on to the roots about 14 days later – at the end of the first week of April. At this time the average soil temperature

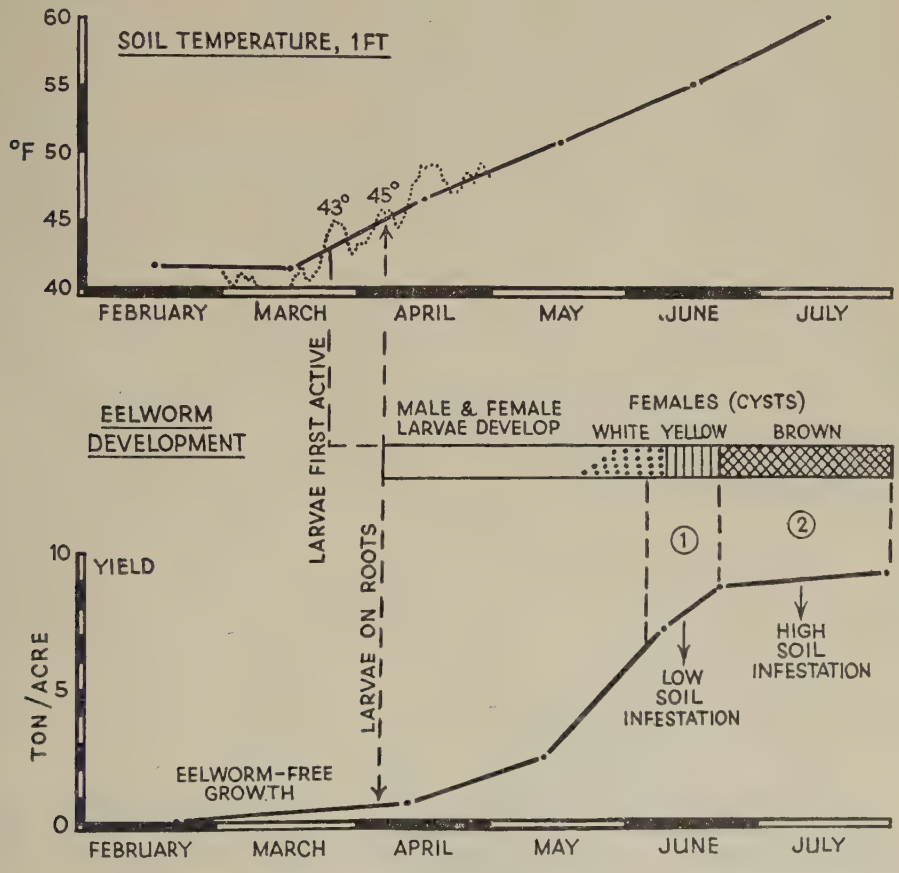


Figure 2. Comparison of soil temperatures with the development of the potato-root eelworm, *Heterodera rostochiensis*, and with the tuber-production cycle (yield) of early Epicure potatoes in Ayrshire. Lifting periods for the crop numbered 1 and 2 are mentioned in the text. The continuous line in the upper graph is plotted from monthly means at Auchincruive; the dotted curve is transcribed from daily values during March and April.

is approximately 45°F, and we have long assumed this value to mark the beginning of eelworm attack on the potato plant. Growth of the host had then, however, progressed so far that there was measurable tuber formation. The crop had indeed about 40 days' eelworm-free growth, or 29 per cent of the total growth cycle of some 140 days.

Cysts began to change from white to yellow about 10 June, and became brown about 20 June. The first-early crop was lifted round 9 June, and the best yields on eelworm-infested ground were just over 7 ton/acre. Land on which crops are consistently lifted year by year at this early date (in period 1, Fig. 2) have low soil infestations of not more than 2 cysts with contents per gram, and frequently only 1 cyst/g. Towards the end of June, yields had risen to about 9 ton/acre, and cysts on the roots were brown. Crops for seed, grown until the end of July, yielded little more, but fields lifted year by year at these later dates (in period 2, Fig. 2) have heavy soil infestations of 4 cysts/g and more. This result links directly with that of Fig. 1, and Fig. 2 further shows that the possibility of obtaining a reasonable yield before the cysts turn brown depends on the delay in starting the eelworm's life cycle until the host has made considerable growth. A differential action of soil temperature upon host and parasite therefore brings important agricultural and pathological effects.

There is also an economic difference between lifting at periods 1 and 2 of Fig. 2. Prices per ton of potatoes are frequently about £30 in early June, diminishing to about £15 at the end of the month. The cash yield from 7 ton/acre in early June thus considerably exceeds that from 9 ton/acre at the end of the month.

The best way of demonstrating that the initial eelworm-free growth of the first-early crop accounts for the lower loss in yield from eelworm attack is to grow healthy and diseased crops at a later period and compare their yields with similar crops grown earlier. This is not easy to do, since for any such comparisons of yield it is necessary to have the same soil with and without the parasite. In 1950 this condition was met by taking cysts from a ton of eelworm-infested soil and mixing them with a ton of eelworm-free soil, another ton of the latter being used as a control. For the first-early crop the comparison between eelworm-infested and eelworm-free conditions cannot be stated with such accuracy. The best approximation comes from an experiment in 1951 on eelworm control using D.D. (dichloropropane-dichloropropylene mixture). This particular control came so near to eliminating the eelworm that there was only one cyst produced per 10 in. of root as compared with 44 on the control. Extensive experimental data show that an infestation on the root of only one cyst/10 in. brings a yield some 15 per cent less than that of a completely healthy plant. The yield of the treated plot was therefore increased by 15 per cent to give the nearest approximation possible to the eelworm-free yield within the totally infested eelworm belt in Ayrshire. The experiment on control was, moreover, of such a long-term nature that any direct stimulating effect of D.D. on the potato crop would be eliminated. Table 1 compares these two results for 1951 and

TABLE 1. DIFFERENCES IN YIELD BETWEEN EELWORM-FREE AND EELWORM-INFESTED EPICURE CROPS GROWN EARLY AND LATE

Year	Potatoes planted	Approx. date 43°F attained in soil	Period of eelworm-free growth		Total growth cycle	Crop lifted	Yield, ton/acre		% loss of yield on infested soil
			Days	% of total cycle	Days		Eelworm-free	Eelworm-infested	
1951	Feb. 27	Apr. 21	53	43	123	Jun. 30	8·46*	5·84	31
1950	May 24	Mar. 24	None	0	107	Sep. 8	7·29	1·49	79

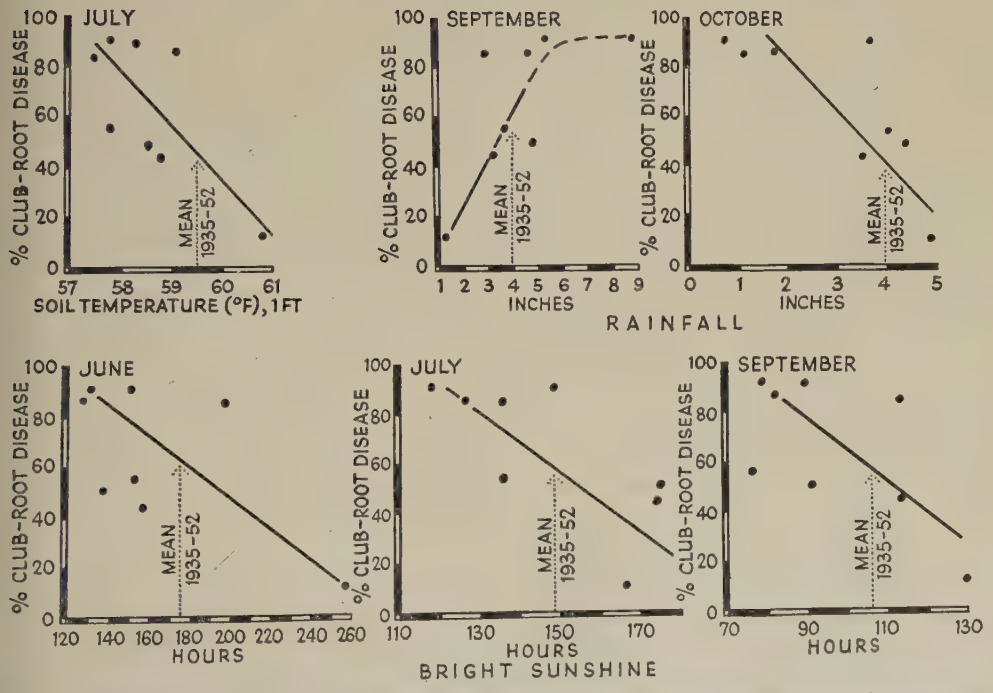
* Calculated value; see text.

50. It shows that the early-planted crop in 1951 had a long initial period of eelworm-ee growth, and had 31 per cent loss of yield on the eelworm-infested land. The crop 1950, on the other hand, was planted so late that there was no period of eelworm-free owth and the loss of yield was 79 per cent.

The initial period of eelworm-free growth brought about by the differential action soil temperature on host and parasite therefore accounts for the low loss in yield from lworm sustained by the first-early crop. It also allows the lifting of a satisfactory eld before the cysts become brown. Lifting while the cysts are white or yellow brings relatively low soil infestation. The practical suppositions mentioned above are therefore nfirmed. If the spread of potato root eelworm should, as seems likely, curtail the reage of ware crops, it would still be possible to maintain *some* native potato supplies growing Epicure crops on relatively frost-free localities early in the year. The increasing ossibilities of direct control of the eelworm, now being investigated, would bring a rther advantage from such a practice, for the concomitant low infestations are more nenable than higher ones to such control.

3. Club-root and climate

The foregoing example considered the differential effects of temperature upon host nd parasite; we next consider the effects of a wider range of climatic factors. The epartment of Plant Pathology at Auchincruive has for some years maintained a series ' disease phenology plots ' grouped round a Class III meteorological station. A standard nge of crop varieties is grown in randomized replicated plots. Cultural treatment is andardized, yields are measured, and the amounts of disease estimated. For club-root



igure 3. Correlation diagrams for significant and near-significant comparisons between the amounts of ub-root disease and monthly means of soil temperature (July), rainfall (September and October), and ight sunshine (June, July and September); 1946-1953. Values of the correlation coefficients are given in Table 2.

disease of turnips and swedes, caused by the fungus *Plasmodiophora brassicae*, all the roots are pulled, and each one relegated to a particular category, from 0 to 10. Category 0 is healthy, category 1 has 10 per cent of its surface area diseased, category 2 has 20 per cent, and so on. This distribution into categories is made from a ' standard area diagram ' (Grainger 1949). The number of roots in each category is multiplied by the number of the category, and the sum of these products is expressed as a percentage of the total number of roots multiplied by ten. This gives a percentage disease rating, which takes into account the amount of disease on each plant and also the amount in the crop. Eight years' results from 1946 to 1953 with club-root upon fully-susceptible varieties of swedes have been examined. This disease is soil-borne, so that soil temperature, air temperature, sunlight and rainfall could conceivably affect either or both members of the host-fungus relation at any stage of growth. Monthly averages for each factor from May to October have therefore been compared separately with the amount of club-root disease occurring when the crop is lifted in November. Relations with air temperature at 4 ft were so random that they were not worthy of further calculation, but Fig. 3 illustrates the significant and near-significant correlations which have emerged between the disease and soil temperature at 1 ft, sunshine and rainfall. Detailed values of correlation coefficients are given in Table 2.

TABLE 2. CLIMATE AND CLUB-ROOT DISEASE, 1946-53 ON SUSCEPTIBLE VARIETIES OF SWEDE. Correlations between the amount of club-root disease and three climatic factors.

	Soil temperature at 1 ft			Bright sunshine			Rainfall		
	Correlation coefficient r	Range of P by t-test	Significance	Correlation coefficient r	Range of P by t-test	Significance	Correlation coefficient r	Range of P by t-test	Significance
May	− 0·445	0·3-0·2	–	+ 0·091	0·9-0·8	–	+ 0·157	0·8-0·7	–
June	− 0·126	0·8-0·7	–	− 0·644	0·1-0·05	s	+ 0·065	0·9-0·8	–
July	− 0·727	0·05-0·02	S	− 0·736	0·05-0·02	S	+ 0·399	0·4-0·3	–
Aug.	− 0·111	0·8-0·7	–	+ 0·190	0·7-0·6	–	− 0·477	0·3-0·2	–
Sept.	− 0·453	0·3-0·2	–	− 0·628	0·1-0·05	s	+ 0·669	0·1-0·05	s
Oct.	− 0·276	0·6-0·5	–	+ 0·246	0·6-0·5	–	− 0·709	0·05-0·02	S

s = near-significant; S = significant. Comparisons of the amount of disease with air temperature from May to October were so random that values of the correlation coefficient were not calculated.

These correlations mean that a *low* incidence of club-root disease on varieties of swede susceptible to the disease, is, at Auchincruive, associated with :

(i) A relatively high soil temperature in July.

(ii) Relatively high amounts of bright sunshine in June, July and September.

(iii) A relatively low rainfall in September.

(iv) A relatively high rainfall in October.

4. DISCUSSION

The two cases considered in this paper show that climatic factors affect the severity of attack by disease by virtue of their action upon both host and parasite. Soil temperature fixes an initial limit to the development of the root eelworm parasite without imposing

such a drastic imposition upon the first-early Epicure potato host. This is an important example of a type of action already known, and of what plant pathologists have called, although usually in other connections, a mechanism of 'disease escape.' The second example of club-root disease is in a different category, for among other relations, an effect of sunshine upon the amount of disease has been demonstrated. This cannot be a direct effect upon the activity of the non-chlorophyllous fungal organism inhabiting the soil. It would seem to be rather an effect upon growth of the host, unless it reflects merely the interaction of some other meteorological factor. This, however, is unlikely, for reasons given below.

The method of calculating correlation coefficients for relations between the amount of club-root disease and monthly averages of climatic factors is not exhaustive. Calendar months may not provide the best time intervals, and the correlations must inevitably be considered in relation to the Auchincruive monthly mean values of the climatic factors which are given in Fig. 3. The important fact is that significant correlations *have* emerged. Most important of all is the significant correlation between bright sunshine and disease. This might, as mentioned above, be an indirect effect, as for example, the effect of solar radiation in raising the soil temperature. The significant correlations in July between disease and soil temperature, and disease and sunshine (Table 2) would be in line with this view. Near-significant correlations of disease with sunshine in June and September, however, are not paralleled by any correlations of disease with soil or air temperature. The correlations with sunshine in June and September are too near significance to be ignored, and the value of the correlation coefficient for disease with soil temperature in June is so low ($-0{\cdot}126$) that it *can* be ignored. In September the correlation coefficient for disease with soil temperature is higher ($-0{\cdot}453$) but still lower than the value of $-0{\cdot}628$ for disease with sunshine. Similar reasoning would also eliminate any effect of sunshine in June through any inter-relation of rainfall and sunshine. We may therefore conclude that, in June at least, there is a correlation between bright sunshine and disease which does not seem to reflect any relation with the other climatic factors considered in this work; it is, in fact, most likely to be an effect upon growth of the host. The observed correlations of soil temperature and rainfall with disease could, on the other hand, be direct effects of climate upon the parasite.

It is not yet clear why there should be correlations of disease with sunshine during the months of June, July and September, but not in August ($r = +0{\cdot}19$; P is between 0·7 and 0·6). Nor is it yet easy to see why the correlation coefficients for disease with rainfall should be positive for September and negative for October. Perhaps it should not, however, be very surprising when we are studying climatic factors which can affect the activity of both host and parasite. Correlations could reasonably be of one sign when they are concerned more with one member of this dual relationship, and of the opposite sign when the other member is involved.

That a factor which affects the growth of a host plant should also have an effect upon the amount of disease, is not now surprising. The writer has a large amount of data, as yet unpublished, which shows a definite relation between the amount of disease and the amount of carbohydrate coupled with the amount of growth. This has been found for three host-fungus relationships, and demonstrates that a physiological balance within the host plays a large part in determining the relative susceptibility of the plant to fungal disease. The finding that the amount of bright sunshine can affect the amount of soil-borne club-root disease contracted by a swede turnip would be directly in line with this new hypothesis. It also shows that in studies of climate and disease, it is necessary to assess the possible effects of all climatic factors, and particularly those which could affect the host.

Acknowledgments

The writer wishes to acknowledge gratefully the help of members of his staff at Auchincruive, and particularly of Mr. David McAughtrie, who is responsible for work on the disease phenology plots, and of Miss A. Kirkwood for the meteorological readings.

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A three-front model for synoptic analyses

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Summary

A model for synoptic frontal analyses derived from frontal-contour-chart studies is described. Three frontal systems named the polar front, the maritime arctic front and the continental arctic front are necessary to portray adequately the thermal-frontal structure of the atmosphere. In-phase wave development on at least two of the frontal systems occurs during major cylogenesis. The three-front (or four-air-mass) cyclone is verified by aerological analyses while the classical occlusion model is not usually observed.

1. Introduction

Studies of frontal contour charts made in Canada since 1945 (Crocker, Godson and Penner 1947; Godson 1950; Anderson, Boville and McLellan 1953) suggest some inadequacies in classical frontal and air-mass concepts. It has been found that a three-front (four-air-mass) model is usually operationally adequate for the winter season for North America. Two frontal systems normally participate in the development of major cyclone waves.

2. Air mass and frontal model

Four main air masses affect North America in the winter. Using the Bergeron classification, these are conveniently named *mT*, *mP*, *mA* and *cA*.

Typical air-mass properties are given in Table 1. The temperatures shown in Table 1 represent a composite picture for middle latitudes taken from frontal contour analyses. They are not statistical values. Indeed, since air masses show considerable variation depending on the strength and amplitude of the westerlies and since frontal surfaces are always present, it is doubtful whether statistical values can give a true synoptic description.

The fronts separating these air masses are :

1. The polar front (*mT*-*mP*).
2. The maritime arctic front (*mP*-*mA*).
3. The continental arctic front (*mA*-*cA*).

TABLE 1. Typical air-mass properties

	Winter				Summer			
Air mass	*mT*	*mP*	*mA*	*cA*	*mT*	*mP*	*mA*	*cA*
P (mb)	(°C)	(°C)	(°C)	(°C)	(°C)	(°C)	(°C)	
850	15	6	− 4	− 25	18	14	2	has been
700	4	− 3	− 16	− 25	8	1	− 10	modified
500	− 15	− 23	− 36	− 40	− 9	− 17	− 28	to colder
400	− 27	− 36	− 48	− 46	− 19	− 28	− 42	maritime
300	− 42	− 53	− 50	− 45	− 35	− 44	− 48	arctic over North America
Tropopause (mb)	200	275	350	400	125	250	300	
θ_w range for mid-troposphere (°C)	13–18	8–12	2–7	< 2	> 15	10–14	4–10	< 4

θ_w = wet-bulb potential temperature

The polar front is usually continuous (at least at upper levels) around the world and is a dynamical feature of the structure of the westerlies (McIntyre and Lee 1954). Exchange processes take place across it through the cutting-off of cold domes (cold lows) south of the westerlies and the cutting-off of warm pockets (warm highs) to the north of the westerlies.

The maritime arctic front is the result of the intrusion of freshly modified cold maritime arctic air masses into the westerly stream. Sometimes the maritime arctic front and polar front exhibit an almost in-phase relationship and are near to one another. A very intense front (polar front) then exists which has a double structure as can be easily demonstrated by aerological studies.

The continental arctic front is largely due to, and maintained by, the ocean – continent (or water-surface snow or ice surface) thermal contrast. In the summer the front disappears or retreats to the ice cap. Two fronts then adequately portray the thermal pattern although incursions of a fourth air mass (or third front) in the form of an unstable and very cold air mass (i.e. a cold maritime arctic air mass) sometimes occur.

3. Synoptic implications of the three-front model

(a) *The three-air-mass cyclone*

Two fronts participate in the development of a major frontal cyclone. An in-phase relationship between two fronts as indicated by a frontal contour analysis is highly favourable for development.

Fig. 1 indicates the polar-front model of a cyclone. The trough of warm air aloft to the north of an occluding frontal wave is a well-defined line of weather discontinuity and it is frequently necessary to indicate this line on a surface frontal analysis. It marks the crest of the wave on successive frontal contours or the surface projection of the trough of warm air aloft. It is called a ' Trowal ' and marked, as indicated in Figs. 1 and 2, on Canadian Weather Charts.

Fig. 2 illustrates a two-front or three-air-mass cyclone. Aerological analyses have indicated that this is the more usual distribution of air masses aloft in a cyclone.

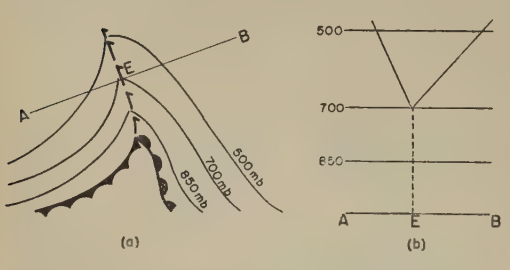

Figure 1. Polar-front model of a cyclone wave. (a) Frontal contours, (b) vertical cross section.

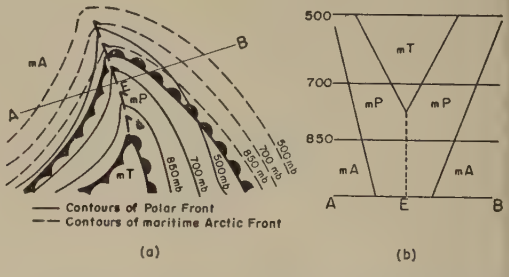

Figure 2. Two-front model of a cyclone wave. (a) Frontal contours, (b) vertical cross section.

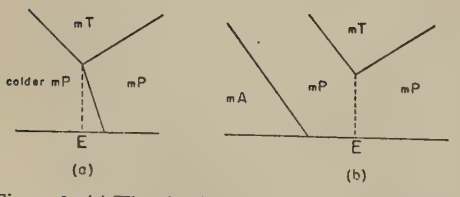

Figure 3. (a) The classical cold-type occluded front, vertical cross section; (b) synoptic representation by a two-front model.

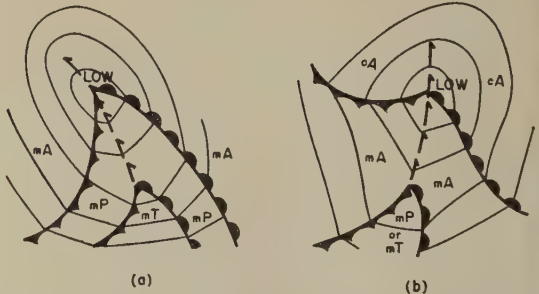

Figure 4. (a), (b) The three-air-mass cyclone.

(b) *The occluded front*

Cases of true occluded fronts as in the classical model are extremely rare over North America nd adjacent regions.

Fig. 3 (a) illustrates the polar-front occlusion as in the classical model. Fig. 3 (b) illustrates he frontal cross section as it is usually observed, and as it would appear in a three-air-mass yclone.

Aerological frontal contour analyses show consistently that the trough of warm air aloft point E) passes a station before the frontal cooling (e.g. from *mP* to *mA*) occurs at low levels Fig. 3 (b)). The model of the cold-type occlusion (Fig. 3 (a)) would require the low-level cooling o precede (or be almost simultaneous with) the passage of the trough of warm air aloft.

The surface cyclone using the polar and maritime arctic fronts is shown in Fig. 4 (a) or with he maritime arctic and continental arctic fronts in Fig. 4 (b). An occluded front based on urface analyses might appear convincing when the *mA* cold front is near to the trowal in the entre of a cyclone (Fig. 4 (a)). This frontal structure, however, does not have the three-dimensional characteristics for an occlusion as in Fig. 3 (a) but conforms to the three-dimensional icture in Fig. 3 (b). An occluded front which can actually be verified by aerological data is ne of the rarer meteorological phenomena. The trowal however exhibits surface weather discontinuities and plays an important role in synoptic analyses without, however, the air-mass movement) restrictions placed on the occlusion.

(c) *Cyclogenesis*

While waves may develop independently on either the polar, maritime arctic or continental rctic fronts, most of the major cyclones occur as a result of in-phase wave development on two ronts. That is, at least three distinct air masses are involved in the development of a wave yclone as indicated in Figs. 4 (a) and 4 (b). Fig. 4 (b) is typical of the development of Alberta ows, the more complex of which are four-air-mass cyclones with all three fronts partaking in he development. The wave on the *cA* front, which is so sharp at the surface, does not appear o be the major cyclogenetic factor since a development of the Alberta-low type always requires he presence of *mT* or *mP* air at a very low level (700 mb or lower). In terms of our model, a rough of warm air aloft (i.e. a trowal) initiates the wave development and cyclogenesis. Wave formations can frequently be foreseen by the use of frontal contour analyses.

4. Conclusions

A three-front model has proved its value in routine synoptic analysis and weather forecasts. Analyses based on this model have shown three-dimensional and time continuity.

Synoptic meteorologists must discard analysis models and concepts which are not consistent with a three-dimensional synoptic-aerological analysis. Frontal analyses based on a two-dimensional study of local weather discontinuities are of little practical use in the broader synoptic problems.

Canadian experience with frontal contour charts indicates that a three-front model is usually adequate and is consistent with aerological observations. Local analysis complications (e.g., secondary fronts or other frontal systems) usually have only local significance and are not continuous in time. Such problems need not be explained by the basic analysis model.

Acknowledgment

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551.594.22

The fine structure of natural point-discharge currents

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1. Introduction

Since Franklin's experiment meteorologists have known that in disturbed weather the electrostatic field near the earth's surface can be intense enough for point-discharge currents to flow from raised points. C. T. R. Wilson (1925) was the first to suggest that these point-discharge currents might be extremely important in the exchange of electricity between the earth and the upper atmosphere, and this suggestion has been investigated and confirmed by many subsequent workers in atmospheric electricity. Point-discharge is also a familiar phenomenon to electrical engineers, who usually call it corona discharge. Loeb and his colleagues in particular have done a great deal of work on discharge in a point-to-plane gap for air at atmospheric pressure.[2] While meteorologists have been mainly concerned with the magnitudes of point-discharge currents, electrical engineers have investigated in some detail the mechanism of the discharge,[3] and they have shown that the current is often made up of a series of pulses.

It is the purpose of this paper to show that many of the effects observed in laboratory experiments on corona discharge are apparent in natural point-discharge in disturbed weather, and that much of the information gained by electrical engineers may be applied directly by meteorologists to natural point-discharge.

2. Experimental arrangements

The investigations were carried out at the Meteorological Laboratory, which is situated on the outskirts of Cambridge.

A number of platinum points with hemispherical tips of various radii were made. Each point was soldered to a brass rod which could be mounted at the top of a pole near the laboratory, at a height of 7 m. The point was connected by a coaxial cable to earth through a resistance of 1 MΩ, and the discharge current from any one of the points was examined by measuring the voltage developed across this resistance with a Cossor double-beam oscilloscope type 1049. A field mill giving an alternating voltage output at 75 c/s, with amplitude proportional to the natural electrostatic field near the earth's surface, was installed outside the laboratory 8 m from the base of the pole. The output of the field mill could be applied to the second beam of the oscilloscope to give simultaneous records of the earth's field and the point-discharge current.

The points could easily be transferred from the pole to a point-to-plane gap in the laboratory. Potentials of up to 50 kv could be applied to the plane for the laboratory experiments. Most of the records in this paper were obtained using a point with a hemispherical tip of radius 0·05 mm.

3. Experimental observations

Discharges in air at atmospheric pressure have been examined in the laboratory by many workers using point-to-plane gaps, and excellent text-book accounts exist (e.g., Meek and Craggs 1953). During the present series of experiments the main results obtained before have been confirmed. In general, for all points, the magnitude and form of the point-discharge currents is governed by the electrostatic field distribution in the immediate vicinity of the point. The material of which the point is made is unimportant provided that the points have been 'conditioned' (Loeb, Kip, Hudson and Bennett 1941).

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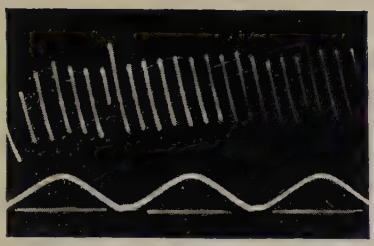
1 (ii)



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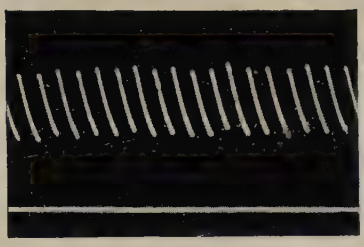

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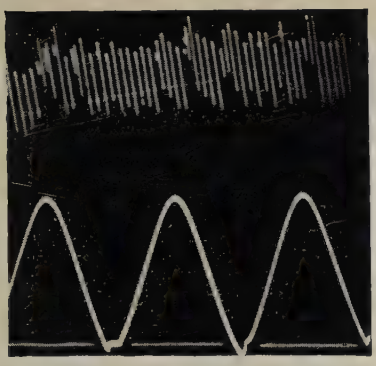
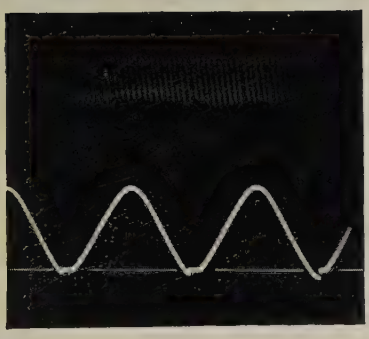


2b (i)

2b (ii)



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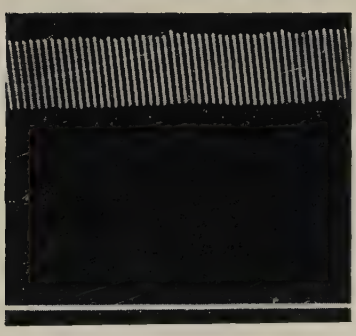

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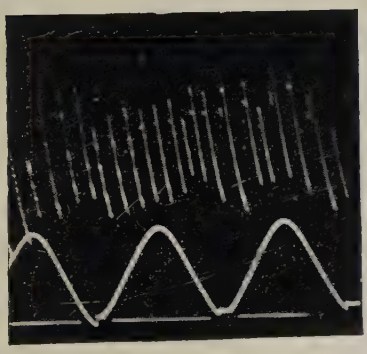
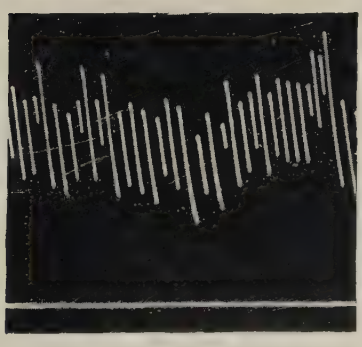


3b (i)



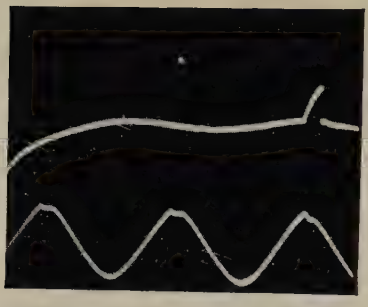
3b (ii)

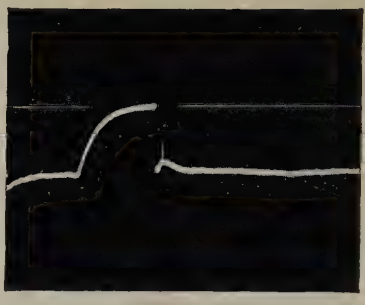
Figures 1, 2 and 3. Examples of point-discharge from a negative point, showing the Trichel pulses.

4 (i)

4 (ii)

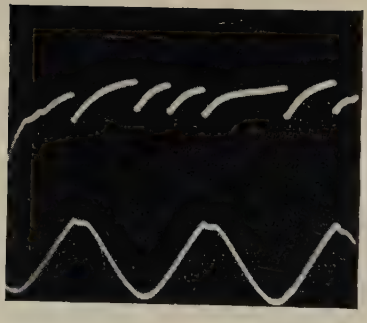
5a (i)

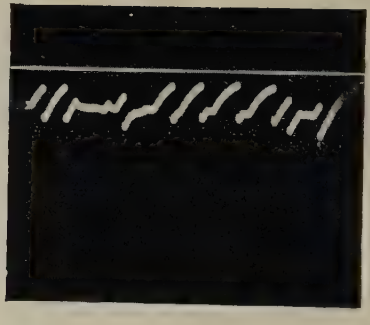




5a (ii)

5b (i)





5b (ii)

6

Figures 4 and 5 are examples of point-discharge from a positive point. Figure 6 shows television interference caused by point-discharge from the television aerial (the horizontal white line is interference from a local ionospheric transmitter).

Vol. **81**, 1955. Plate II.

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In a point-to-plane gap with the point as the negative electrode the current normally consists of a series of regular pulses. These were first described by Trichel (1938). Each pulse carries about the same ' quantum of charge ' from the point, and a change in the corona current occurs as a proportionate change in the repetition rate of the pulses. Occasionally there is a spontaneous ' change of regime ' with a visible alteration in the position of the active discharging spot on the point, and a change in the frequency of the pulses. This does not necessarily produce a change in the corona current, for the change of pulse frequency is often associated with a reciprocal change in the amplitude of the individual pulses.

When the point is positive the picture is more complicated. At the onset of appreciable current there is a region sometimes called the Geiger-counter region where ' inductive kicks ' occur. These are associated with incipient streamers from the point. Inductive kicks may initiate small and irregular ' burst pulses,' but they do not always do so. As the potential across the gap is increased the inductive kicks disappear and the current becomes more or less steady, with slight irregularity due to ' self-sustained burst pulses ' (see Meek and Craggs 1953).

Natural discharge from the same points as were used in the laboratory experiments, but now mounted on the pole outside the laboratory, was observed during showers and thunderstorms in the spring of 1954. Although rapid variations were often produced by overall changes in the electrostatic field, formation and dissipation of space charges by rain and wind, weathering of the point and so on, it was soon apparent that all the main effects seen in the laboratory were reproduced in natural conditions. These are illustrated in the original records that follow.

4. Explanation of the plates I and II

In the plates the time scale is the same for all the records, and is about 30 millisec for the complete sweep. The horizontal line in some of the records is the point-discharge current zero.

Figs. 1 and 4 show how the ' input time-constant ' affects the recorded form of the Trichel pulses. Fig. 1 (i) is a typical negative-point Trichel discharge, obtained in the laboratory using a short input cable; and Fig. 1 (ii) shows the effect of connecting the long coaxial cable to the outside point across the input. The extra cable increases the input time-constant, thereby lengthening the decay time of the pulses as well as reducing their amplitude. Fig. 4 (i) shows positive-point discharge in the inductive kick region; and again in Fig. 4 (ii) the modifying effect of the long cable. Fig. 4 (i) shows some inductive kicks which initiate small burst pulses and some which do not. When the long cable is connected the small burst pulses can hardly be distinguished but the inductive kicks can be recognized quite easily in 4 (ii).

The records on the left-hand side of Figs. 2, 3 and 5 show some of the forms taken by point-discharge currents in natural point-discharge. The right-hand figures are similar, but were obtained with the point-to-plane apparatus in the laboratory. In order that the natural and laboratory records might be strictly comparable, the coaxial cable to the outside point was left connected when the laboratory records were being made, and the same point was used. The lower trace on the left-hand figures is the 75 c/s output from the field mill which measures the field at the ground near to the point.

Figs. 2a (i) and 2b (i) were taken in quick succession and show a change of field with an associated change in the point-discharge current. The field is positive according to the usual convention in atmospheric electricity (i.e., directed downwards) and the point is consequently negative. The pulses in these figures are quite typical negative-point Trichel pulses. The change of field between 2a (i) and 2b (i) is from + 3·5 v/cm to + 16·0 v/cm and the change in the point-discharge current is from 0·1 μA to 0·2 μA with a frequency change roughly in proportion, from 750 pulses/sec to 1,700 pulses/sec. A similar change in corona current was produced in the laboratory (Figs. 2a (ii) and 2b (ii)) by increasing the potential on the plane of the point-to-plane apparatus.

Figs. 3a (i) and 3b (i) show a ' change of regime ' in which the frequency and size of the pulses changed spontaneously, while the field remained constant at + 10 v/cm and the mean point-discharge current did not change very much, from 1·6 to 1·5 μA. The right-hand Figs. 3a (ii) and 3b (ii) show a similar spontaneous change of regime for the point in the laboratory.

Fig. 5a (i) is a record of point-discharge, near onset, in a negative field of − 7·8 v/cm and 5b (i) shows a typical discharge in the inductive kick region. Figs. 5a (ii) and 5b (ii) are similar laboratory records of positive point-discharge. Inductive kicks are rarely seen in natural point discharge. This is not surprising as the voltage region in which inductive kicks occur is very narrow.

5. The relation between point-discharge current and field

The point-discharge current I has often been used as a measure of the field F at the earth's surface using empirical relations such as

$$I = a(F^2 - M^2) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

given by Whipple and Scrase (1936); a and M are constants. The present techniques enabled the field as recorded by the field mill to be compared simultaneously with the point-discharge current using the high time resolution of the oscilloscope. It was soon apparent, on the basis of this comparison, that over short time intervals relation (1) was by no means obeyed (for example in Fig. 2 a five-fold increase in field is accompanied by only a doubling of point-discharge current). Occasions were not uncommon, particularly after a lightning discharge, when the field at the ground was for a few seconds opposite in sign to that indicated by the point-discharge current. These departures from relation (1) may be partly accounted for by the formation of temporary space-charges between the ground and the discharging point, and partly by the fact that much of this work was done near point-discharge onset; the results on which Whipple and Scrase based (1) were very scattered for near-onset point-discharge currents.

6. The measurement of point-discharge currents

Previous workers studying point discharge have assumed that the current is continuous. Considerable error may be introduced in the measurement of pulsed point-discharge currents if a non-linear current measuring device is used. As an example consider the error introduced by the use of an instrument which responds to the root-mean-square value of the current.

A typical Trichel-type discharge current may be represented by a series of pulses of frequency f. Each pulse has the form of an instantaneous rise of current to a value i_0 followed by an exponential decay with a time constant T. For a discharge of this type the mean current is given by

$$I_{\text{mean}} = \frac{\int_0^{1/f} i_0 \exp(-t/T)\, dt}{1/f} \simeq i_0 T f \quad . \quad . \quad . \quad . \quad (2)$$

assuming that the time interval between pulses is long compared with the decay time constant. And the root-mean-square current is given by

$$I_{\text{rms}} = i_0 \left\{ \frac{\int_0^{1/f} \exp(-2t/T)\, dt}{1/f} \right\}^{\frac{1}{2}} \simeq i_0 \left(\frac{Tf}{2}\right)^{\frac{1}{2}}, \text{ if } 1/f > T \quad . \quad (3)$$

Thus, if the time constant associated with the discharging point is very small, a non-linear instrument can give a very misleading idea of the mean point-discharge current. If the pulse frequency were $f = 1{,}000$ pulses/sec and the decay time constant were $T = 10^{-5}$ sec, Eqs. (2) and (3) show that the root-mean-square instrument would be in error by a factor of about 7.

The time constant of decay, T, is purely instrumental. If the point-discharge currents are passed through an integrating circuit before measurement, the difficulties treated in this section are avoided.

7. Television interference

An interesting form of television interference is almost certainly caused by the pulses of natural point-discharge currents (Strafford 1948). In stormy weather a peculiar 'snow-storm' pattern is often seen on television screens. This is caused by each discharge pulse producing a brilliancy modulation. With experience, an observer can judge the approximate pulse-repetition frequency, and follow the variations of the natural electrostatic field in the vicinity of his home. Fig. 6 shows an example of this kind of interference; it was produced by inducing negative-point Trichel discharge from a television aerial by means of an artificial field. The pulses appeared extremely brilliant on the screen. They have defocused the electron-spot in much the same way that ignition interference sometimes does. The apparent doubling of the spots is instrumental, being caused by poor aligning of the tuned circuits of the television receiver.

The interference on television sound is usually a gentle hissing, but sometimes a quiet whining is heard, which changes in pitch with the variations in the 'snow-storm' pattern on the screen.

8. Conclusion

It has been shown that most of the phenomena observed in the laboratory also occur in natural point-discharge. No attempt is made to claim that the point-to-plane apparatus available in the laboratory in any way simulates the natural configuration of earth and atmosphere. Since, however, the discharge from a point depends primarily upon the field distribution in its immediate vicinity, the relevant parts of many field distributions encountered in nature can be reproduced with comparative ease in the laboratory.

The continual variations in natural conditions make it necessary to make a considerable number of observations in order to derive a comprehensive pattern for the overall behaviour in all kinds of weather, and the work is being continued at Cambridge.

Acknowledgments

We thank Dr. T. W. Wormell for his helpful advice in the preparation of this paper, and Mr. R. V. E. Brundish for his assistance with the photography. We are indebted to the British Broadcasting Corporation for permission to publish a photograph of one of their Television transmissions, and one of us (M.I.L.) thanks the Department of Scientific and Industrial Research for a maintenance grant which made it possible for him to do this work.

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551.311.18 : 551.583

Arctic ice and weather relationships

By I. I. Schell

May I be permitted at this late date to comment on an earlier paper by Sir Gilbert Walker (1947) in your *Journal* on the question of a relationship between the ice in the N. Atlantic-Arctic and the weather of northern Europe, especially as this question was raised recently again (Frankcom 1953) ?

Walker concluded that, in general, relationships of the arctic ice with previous weather conditions are closer than the relationships of the ice with subsequent conditions, and with this there can be no dispute. But if we are properly to assess what the latter may be, it would be helpful to proceed as far as possible from physical considerations rather than let the matter rest on statistical grounds alone; when evidence relevant to the issue in question is placed alongside evidence seemingly bearing no relation to it, there is the danger of obscuring the outlines of the relationships sought.

In accordance with the initial hypothesis, mainly by Wiese (1924), a more southerly extension of the arctic ice in the northern N. Atlantic is associated with a greater outpouring of water from the Arctic, including a stronger East-Icelandic polar current, and, as the colder water penetrates farther south than usual, the zone of temperature-contrasts between the cold arctic and the warm Atlantic water, is also displaced southward. Further, as the mean northern N. Atlantic storm-track may be expected to be closely associated with the zone of convergence of cold and warm waters, with a more southerly extension of the ice, fewer storms and less precipitation may be expected in the higher latitudes and more storms and heavier precipitation in the less northerly latitudes.

Additionally, because of a marked tendency for persistence in the circulation, the relationship associated with a more southerly extension, for example, of the arctic water and ice, may be expected to continue for some time beyond the ice season, and hence, for (1) colder water and air farther south, (2) a more southerly position of the storm track, and (3) fewer storms and lighter precipitation in the higher latitudes, and more storms and heavier precipitation in the less high latitudes, also to follow the ice season.

To show this with respect to (1), the correlation of the April-August ice limit of the Greenland Sea and Barents Sea combined, with the following September-February sea-surface temperatures at Papey, off the east coast of Iceland, is $r = -0{\cdot}64$*, and with the following September-February air-temperature of Iceland (Stykkisholm and Berufjord), $r = -0{\cdot}54$*. Additionally, the correlation of the ice limit with the following September-February sea surface temperature at Thorshavn-Myggenaes to the south-eastward, is $r = -0{\cdot}57$*. No data are readily available with respect to cyclonic frequencies but with respect to (3), the correlation of the amount of ice in the Greenland and Barents seas with the following September-February precipitation of Iceland is $r = -0{\cdot}43$*; with the October-December precipitation of Great Britain and surrounding area (Schell 1947), $r = 0{\cdot}48$* (with an opposite sign for less northerly latitudes.)

Since, in a broader sense, the relationships of the arctic ice with the contemporary and following features of the weather arise from the changes in the ocean and air circulations which determine the arctic ice, we might have preferred to employ the circulation itself rather than its index, the ice; this we are unable to do as we have no means of measuring this circulation in a way which might later yield the same degree of correlation as was obtained with the arctic ice, the correlations of the N. Atlantic oscillation with the following September-November temperatures, for example, having been minimal (Walker and Bliss 1937). Thus, until the actual circulation controlling the following September-February weather in the northern N. Atlantic can be more closely defined, the arctic ice can be viewed in the special role that has been assigned to it by circumstances, a role that has not yet been fully investigated.

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* 1895-1939, 1946-49 ($n = 49$).

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26 September 1954.

551.574.1

The physics of rainclouds

By C. H. B. Priestley

While appreciative of the generally excellent review of the physics of rainclouds which Browne, Palmer and Wormell (1954) have recently given us, I have been concerned at the small space allotted to one of the central problems, the initial stage of growth by condensation on a range of nucleus sizes. Does this reflect a retarded state of development of this aspect of the subject, or the opinion of the reviewers that it is not a very important one ? In particular there is no mention of the recent work of Squires (1952a, b), whose comprehensive and precise analysis allows a considerably more exact approach to the problems associated with this phase than had previously been achieved. While this lends more certainty to some conclusions which were already widely believed it may lead to a subtle but significant modification to others.

For example, Squires finds that, for large (20 μ to 100 μ) droplets to form by condensation there must be either : (i) only rather few giant nuclei present, and cooling must continue for several thousand seconds, or : (ii) no giant nuclei and rather few larger than a considerably lower size-limit, in which case shorter periods may be sufficient. In both cases limits exist to the concentration of the largest nuclei present, *above or below* which, conditions will not favour the formation of non-freezing rain; the favourable combination of conditions is likely to be met at times but not in general. Further, type (i), when rain results, would be short-lived, the giant nuclei being washed out of the air entering the base of the cloud but, given just the right initial nucleus spectrum, this removal could lead to type (ii) so providing a rain process which may not easily start but contains a regenerative element. These may help to explain the greater unreliability of warm rain, as contrasted with rain when the Bergeron process is operative; and the impression that, even with heavy clouds present, rainy or dry conditions tend to persist and be broken only by a comparatively large change in the synoptic situation.

In general Squires's calculations give a fresh emphasis to the critical nature of the nucleus spectrum, as opposed to the concentration of nuclei *per se*, and underline the present paucity of observational data on this aspect.

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28 October 1954.

551.583.14

A close parallel between temperature fluctuations in east Canada and Britain

By G. S. Callendar

In his opening address to the panel on Climatic change at the 1953 Toronto Meteorological Conference, Professor G. Manley referred to the surprising similarity of the temperature fluctuations on both sides of the North Atlantic. This relationship is shown very clearly in Fig. 1, which compares the 10-year moving average of temperature at Kew with that shown by a group of five stations in southern Ontario. The latter is taken from the Canadian temperature series presented to the Toronto Conference by Dr. R. W. Longley (1954).

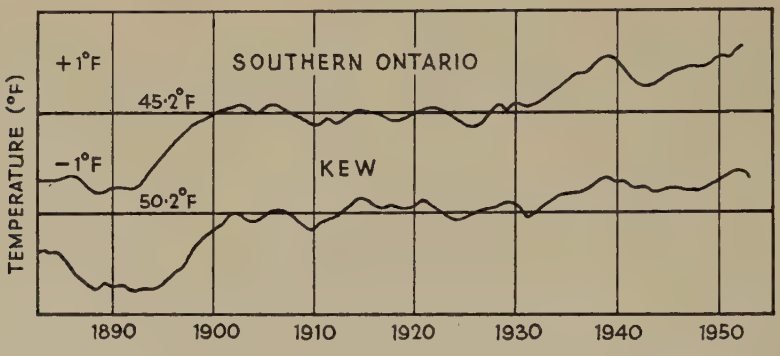

Figure 1. 10-year temperature averages in E. Canada and England, 1873-1952.

When the great distance and also the very considerable difference of climates between the two regions represented by these curves are taken into account, it is truly remarkable that the decadal temperature fluctuations should be so nearly concordant. The same may be said of the other districts in east Canada reported on by Longley, except in the case of northern Ontario where the few early readings may be unreliable. To the west of the great lakes the minor fluctuations are often the reverse of those in the east, especially so in the last few years when severe winters on the Prairies have caused a sharp drop in the Western curves.

The cause of the micro-agreement often shown by these fluctuations in east Canada and Britain would seem to be a matter for the meteorologists who study long-wave patterns in the zonal westerlies.

A further point of interest shown by these temperature observations is that the overall trend has been almost the same in Canada and Britain. The change of average temperature between the first and second half of the period is readily computed for each of the Canadian districts by taking out the decadal means from Longley's curves. This change, averaged for the several districts of east Canada, is here compared with representative British temperature differences between the same two 40-year periods, 1871 to 1910 and 1911 to 1950. For western Canada the district values are only available from about 1890 to cover the whole region, and here the 30-year half periods, 1891 to 1920 and 1921 to 1950, are used. The Canadian values are averages for about 30 stations in each case.

As the general trend of temperatures in northern latitudes of Asia was very similar to that in Canada and Britain between the 1880's and 1930's (Willett 1950), it probably would be safe to conclude that it has been characteristic of most of the north temperate zone in this century. Old British records, with good reliability through much longer periods (Manley 1946, 1953; Callendar 1952) indicate little overall trend during the 19th century, with averages about 0·7°F below that of the last 40 years. In view of these regional temperature trends, it is highly significant that the great majority of glaciers have been receding of late in all climates where they are sensitive to small temperature fluctuations, because this indicates that an upward trend is by no means confined to north temperate latitudes, although doubtless too small to be noticeable in many regions.

TABLE 1.

Area	Temperature change (°F)	Overall rate (°F/yr)
E. Canada	0·82	+ 0·02
W. Canada	0·87	+ 0·03
England (Manley 1953)	0·78	+ 0·02
Scotland (Aberdeen)	0·91	+ 0·023

In this connection it is also significant that recent calculations on the radiative thermal equilibrium by G. N. Plass (1953), indicate that temperatures should rise by about 1°F in certain climates for a 10 per cent increase of atmospheric carbon dioxide, because the latter is approximately the amount (25×10^{10} ton), added by combustion of coal and oil since the turn of the century, and moreover, as we have seen, the rate of rise in certain regions would give 1°F in 50 years. Plass, while noting the possible significance of his results, claims only that they are indicative. Hewson (1954) considers that an important effect of the extra CO_2 could be a fractional reduction of cloud cover, leading to smaller reflection loss from the solar beam, or to an upward displacement of the thermal equilibrium. This supports a recent suggestion by the writer (Callendar 1949).

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16 November 1954.

551.577.37 : 551.578

Three unusual depths of snowfall in Japan

By H. Arakawa

Many people, including the general public, railway and highway maintenance engineers, etc., are very much interested in the depth of snowfall; thus snowfalls of unusual depth receive a great deal of publicity in the press and in weather reports.

Recently a large-scale compilation of depths of snowfall throughout Japan has been made by the writer (*Sekisetsu Ruinen Kisho Hyo*, Central Meteorological Observatory, Tokyo, 198 pp., 1954). Three outstanding values will be described below.

TABLE 1. Details of stations including the record snowfall depths

Station	Elevation (m)	Location	Greatest depth of snowfall (m)	Date	Commencement of records
Ibukisan	1,318	35°25′N, 136°24′E	11·82	14 Feb. 1927	1923
Makawa	1,100	36°32′N, 137°31′E	7·50	26 Feb. 1945	1935
Tochiomata	293	37°11′N, 139°05′E	7·08	15 Feb. 1936	1923

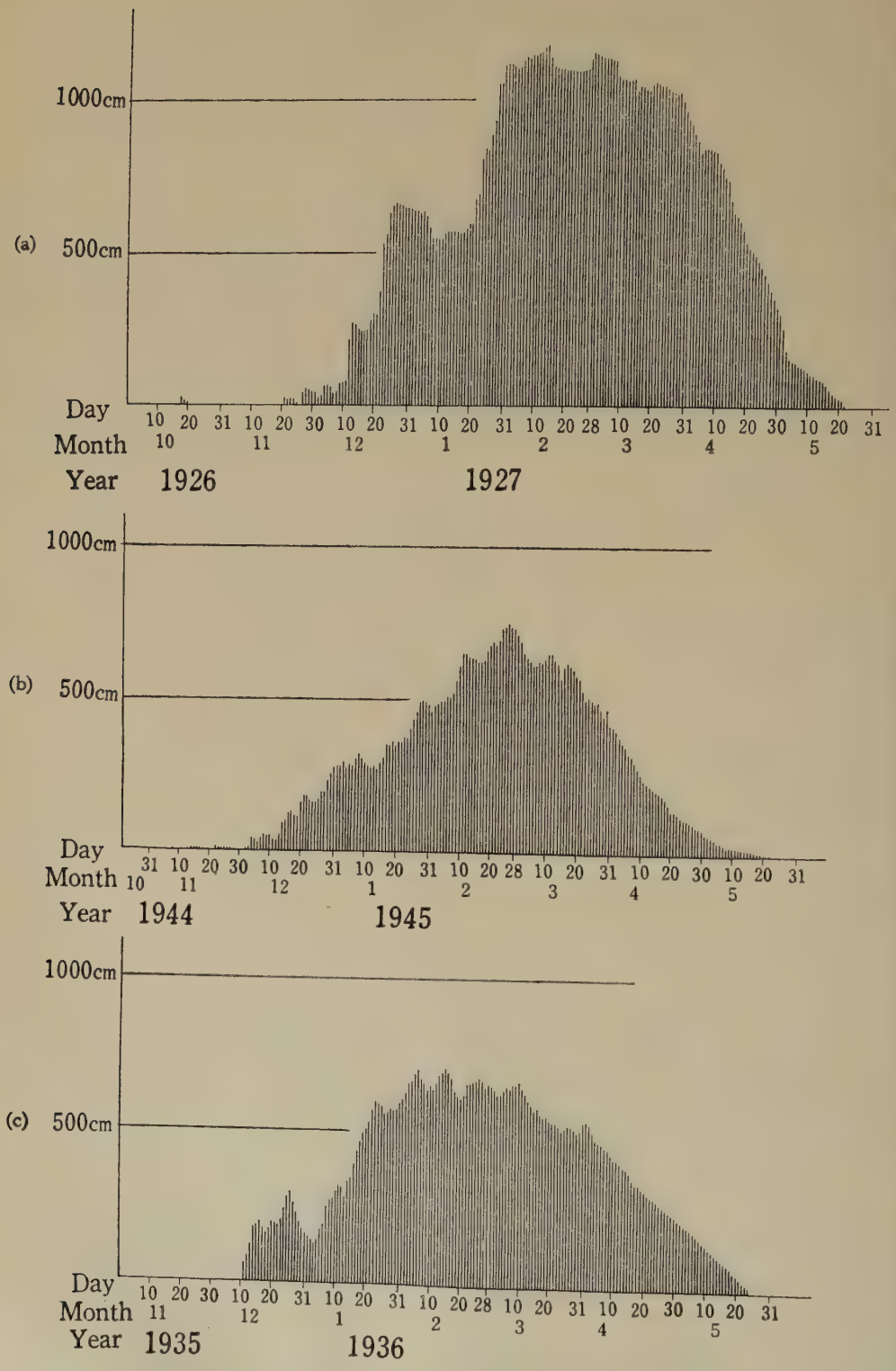

Figure 1. Daily values of depths (cm) of snowfall for (a) Ibukisan, (b) Makawa and (c) Tochiomata.

A depth of snowfall of 11·82 m on 14 Feb. 1927, was reported at Ibukisan Weather station, one of the Government weather stations in Japan. This depth of snowfall, accepted as being the record value for Japan, was measured by well-trained meteorological observers.

Fig. 1 gives the graphical tables for daily values (at 1000 Japanese civil time) of accumulated depths of snowfall during the winters of maximum snowfall.

Meteorological Research Institute,
Tokyo.
16 November 1954.

Edinburgh meeting, 16-17 July 1954

As already reported on p. 648 of vol. 80, two symposia were held at a meeting of the Society in Edinburgh on 16 and 17 July 1954. Details of the papers and the general discussion are given below.

551.510.533

Symposium on the Upper Atmosphere

(Held on 16 July 1954)

Opening Address

By SIR EDWARD APPLETON, F.R.S.

Principal of the University of Edinburgh

SUMMARY

Sir Edward Appleton gave a survey of the present state of our knowledge of the structure of the upper atmosphere, indicating the regions that are the seats of the various phenomena which were to be discussed by subsequent speakers in the symposium. He likened the upper atmosphere to a vast laboratory in which the sun daily actuates a series of experiments at low pressures, in conditions and on a scale which we cannot hope to reproduce experimentally. Solar ultra-violet radiation creates, at different levels, the ozonosphere and the ionosphere by dissociating molecules and by ionizing both atoms and molecules. Further, the gravitational attraction of the sun and moon causes marked atmospheric tides at high levels, and the influence of the earth's magnetic field comes into action as soon as any charged atmospheric particle begins to move. Meteors, sporadic and in showers, produce streaks of ionization which can be observed by eye if clouds are absent and always – even in Manchester – by radio. It may be that fine meteoric dust causes a permanent part of the ionosphere.

Sir Edward then discussed some ionospheric problems which he and his research assistants in Edinburgh, Mrs. Pritchard and A. J. Lyon, have recently examined. Considering first the world morphology of the $F2$ layer, they have investigated the variation of $fF2$ (the critical frequency for the $F2$ layer and an indication of the maximum electron density) at noon at the equinoxes with both geographical and geomagnetic latitude and have found the closer fit with the latter. The curve shows a trough in the region of the equator, where $fF2$ is a minimum. When curves are drawn for other times of the day, it is found that in the late evening hours the trough becomes a crest. No such effect appears in similar curves drawn for the E and $F1$ layers. This difference between the $F2$ layer and the lower E and $F1$ layers is not thought to indicate that the $F2$ layer is caused by electrified corpuscles. Ultra-violet radiation, certainly the cause of the E and $F1$ layers, is also probably responsible for the $F2$ layer and the effect found is due to geomagnetic distortion. The movements associated with the diurnal oscillation, tidal or thermal in origin, which these observations disclose, would be subject to strong control by the geomagnetic field at the low densities obtaining at the $F2$ level. But, though theories have been proposed by Martyn, Maeda and Weiss, there is as yet no satisfactory theoretical explanation. The investigation of the $F2$ layer has further revealed that, during an ionospheric storm, the value of noon $fF2$ is reduced in high latitudes and increased at the equator.

While this examination of the world morphology of the E and $F1$ layers has shown no strikingly abnormal effects, even here there are small departures from simple theory. It was well worth while investigating such discrepancies, for there is always the possibility that the investigation may yield a method of measuring the magnitude of the factor which itself causes the discrepancy. In illustration, some features of the E layer were considered; from the

ontinuity equation, in an ionized medium it follows that :

$$N = \sqrt{\frac{q_0 \cos \chi}{\alpha}},$$

where N is the ion (or electron) concentration, χ the zenith distance of the sun and α the coefficient of recombination. If q_0 and α are constant, then N is uniquely determined by $\cos \chi$. A curve showing the annual variation of N_E for constant χ may be drawn using experimental data, since χ is the same at noon in winter and in the early morning or late evening in summer. It is found, in fact, that N_E for constant χ has a minimum in June, so that either q_0 or α, or both, varies; conditions are not the same in summer and winter at the same height.

Finally, Sir Edward discussed the methods being used in an attempt to determine accurate values of the recombination coefficient at low pressures at all times of the day throughout the year and showed how the coefficient may be determined from observation of the time-lag between the maximum of the ionizing process (at noon) and the ionization maximum; values of the order of 10^{-9} and 10^{-8} have been obtained. The opportunity, afforded by the recent eclipse, of making such measurements at a minimum of solar influence has been utilised; Mr. Lyon has successfully made radio observations on the ionosphere at Slough, and these are at present being analysed.

551.557.551.510.535 : 551.590.25

The atmosphere between 85 and 105 km studied by means of radio-echoes from meteor trails

By I. C. BROWNE

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Summary

Meteors falling into the earth's atmosphere produce, by a process which was described briefly, a trail of ionised air. By studying the reflection of radio waves from these trails (' meteor echoes ') the atmosphere at heights near 95 km can be sampled several hundred times per hour throughout the day, simply and at low cost. The paper summarized results obtained by many workers at Jodrell Bank since 1946 (see references).

The intensities and heights of appearance of meteor echoes lead directly to estimates of air density (about 3×10^{-9} g/cm^3 at 95 km) and of scale height (about 6·5 km at 95 km), while the durations of meteor echoes afford a means of measuring the atmospheric diffusion coefficient. The measurements were compared briefly with independent estimates.

By a special technique, movements of meteor trails of the order of 1 m are detected, and hence winds measured at heights of the order of 95 km. Although ' winds ' obtained by ionospheric methods may represent travelling disturbances, the ' meteor ' winds are true mass movements of the air. It is found that there is usually a prevailing wind of the order of 10 m/sec, upon which is superimposed a semi-diurnal variation which may have an amplitude of 30 m/sec. The phase and amplitude of the semi-diurnal component show seasonal changes. The winds increase with height, the shear having a value of about 2×10^{-3} sec^{-1}, a value comparable with that found in the troposphere, although there is no change in wind direction with increasing height. There is no detectable vertical component of wind.

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Discussion

Dr. T. W. Wormell: Is it quite certain that the motion of an ionised meteor trail is identical in magnitude and direction with the general air motion at the same level? Can we neglect completely the effect of magnetic forces on the moving ions and electrons?

Dr. O. G. Sutton (*President*): The shear in the wind indicated by Dr. Browne's measurements was apparently similar to those observed in the lower regions of the atmosphere, i.e., those arising, in part, from the frictional drag of the surface. Could Dr. Browne suggest a mechanism which would account for the maintenance, at such great heights, of such a regular shear as that shown?

Dr. I. C. Browne (*in reply*): Dr. Wormell has raised the question of the effect of the terrestrial magnetic field on a drifting ionised meteor trail. If the trail lies across the magnetic lines of force, there is no net effect, because the magnetic field produces equal and opposite forces on the positive and negative ions and electrons in the column, these forces being neutralized *in a bounded plasma* by the electrostatic attraction between the ions. The situation is analogous to that of a straight wire conductor of finite length moving across a magnetic field. If the meteor trail lies along the lines of force, there will be some effect, because the ions and electrons will tend to spiral round the axis of the trail, and any subsequent collisions will exert a braking action on the drift motion of the trail as a whole. Rough calculations indicate that this effect will be negligible, however. Thus the drift of a meteor trail, unlike that of an effectively unbounded ionised region, such as a large ionospheric irregularity, represents the mass-movement of the surrounding air even in the presence of a magnetic field.

In reply to Dr. Sutton, I think that the similarity between the vertical wind shears found in the troposphere and between 80-100 km is fortuitous. As Dr. Greenhow has shown from his measurements (see references) the wind between 80 and 100 km is largely tidal. According to Wilkes's theory, the amplitudes of the tidal oscillations in this region are dependent on the temperature structure. This is such that the semi-diurnal pressure variations should increase upwards by about 100 per cent between 85 km and 100 km, and, of course, the tidal wind velocities should also increase upwards by this amount, giving a shear of the magnitude found by Greenhow. The shear in this region may thus be said to have a thermal origin, unlike the tropospheric wind shear, which is largely frictional in origin.

551.510.535 : 523.8 : 538.56

The scintillation of radio stars

By H. P. PALMER

Jodrell Bank Experimental Station, University of Manchester

Summary

This note presents a brief summary of the work of Little, Maxwell and Dagg (1952, 1954) on the fluctuations of the power received from the intense sources of radio emission in the constellations of Cassiopeia and Cygnus.

Receiving equipments, 4 km apart, give almost identical records of the fluctuations, but there is no correlation when the receivers are 200 km apart. The fluctuations occur mainly during the night, the degree of fluctuation varying from night to night, and showing a close correlation with the occurrence of ' spread F ' ionospheric echoes. This evidence suggests that the fluctuations are caused by a terrestrial agency, and not by variations of the power emitted by the radio source. It is thought that the fluctuations are analogous to the ' twinkling ' of visual stars and are caused by a diffracting screen of irregularities above the maximum of ionization in the F region of the ionosphere.

When records from receivers two or three km apart are compared, small time-differences can be detected between the records of similar fluctuations. These may be interpreted as evidence for drift motions of the diffracting screen. Records from a triangularly-spaced set of three stations have recently given both the drift speeds and directions. For observations when the source is near the zenith, in Britain, the velocities are usually in the range 50-300 m/sec, in an east-west direction.

In the winter months, when the stars transit near midnight, the drifts are towards the west before 01 GMT. There appears then to be a fairly sharp reversal, and the drifts are predominantly towards the east thereafter. As the fluctuations rarely occur in the daytime, no other reversal has been observed.

When the stars are observed near lower culmination the line of sight passes through the F region in the auroral zone. Fluctuations are observed at all times (even on magnetically quiet days), and they indicate rather higher velocities, and more variable directions of drift. The drifts are predominantly towards the west in the period 09-21 GMT and towards the SE. or SW. from 21-06 GMT. The apparent N-S components of these drifts might be caused by vertical movements. The appended references contain full accounts of this work.

References

Little, C. G. and Maxwell, A. 1952 'The scintillation of radio stars during aurorae and magnetic storms,' *J. Atmos. Terr. Phys.*, **2**, p. 356.

Maxwell, A. and Dagg, M. 1954 'A radio-astronomical investigation of drift movements in the upper atmosphere,' *Phil. Mag.*, **45**, p. 551.

Discussion

DR. A. E. M. GEDDES : Are there any other constellations besides Cygnus and Cassiopeia which show this effect ? Is the effect shown in the first slide repeated night after night ?

MR. C. A. HALSTEAD : Can Dr. Palmer tell us whether a lower general intensity of radio stars during the day might prevent observation of scintillation at daytime transits ?

DR. H. P. PALMER (*in reply*) : In reply to the first question I would say that the scintillations are only observed easily in the power received from these two intense sources. All other radio sources emit, at these frequencies, less than one-tenth of the power from the source in Cassiopeia, and scintillations are not easily distinguished from statistical fluctuations of the receiver output.

The first slide showed scintillation during the transit of Cassiopeia I. The transit occurs each sidereal day, and scintillations are pronounced during about one night in three.

In reply to the second question, I would say there is no ionospheric absorption at the frequency of 80 Mc/sec used for this work, and this scintillation appears to be a diffraction effect which occurs at night.

Tidal oscillations of the lower stratosphere

By D. H. JOHNSON

Meteorological Office, Dunstable

(See p. 1 of this issue)

551.557 : 551.510.535 : 55.038

Upper winds from geomagnetic observations

By D. H. McINTOSH

Meteorological Office, Eskdalemuir Observatory

Summary

The belief that it may be possible to interpret geomagnetic observations in terms of upper atmospheric winds is based on the theory proposed by Balfour Stewart in 1882 to account for the observed daily magnetic variations. Balfour Stewart's suggestion was that the upper atmosphere is highly conducting and that this conducting air in its daily horizontal tidal movement

across the vertical component of the earth's field induces by dynamo action a varying electric current whose magnetic field is observed at the ground as the magnetic daily variation. The theory of the magnetic solar daily variation, and of a corresponding lunar effect, was developed by Schuster and later by Chapman. All the problems of the theory have not been resolved but its essential correctness has received support in three ways which have led to its general acceptance : the experimental discovery of the ionosphere; characteristics of the magnetic effect which accompanies a large solar flare; and the discovery by a rocket ascent in 1949 of a discontinuity of the geomagnetic field at about 100 km due to the concentration of an electric current there.

The atmospheric dynamo theory of the magnetic daily variations implies that any steady wind in the upper atmosphere must also be reflected in the magnetic records, and during the past ten years Wulf has examined this possibility, with limited success. Recently Vestine, proceeding on the assumption that all currents causing magnetic disturbances are located within the high atmosphere, has derived wind systems compatible with the annual variation of magnetic disturbance and with the features shown by a particular magnetic storm. Although the validity of Vestine's basic assumption is in considerable doubt, his contribution remains an important one and points the way to future progress. The prospect of obtaining actual measurements of the ionospheric winds by means of the newly developed radarsonde, combined with the regular ionospheric measurements now available, make it probable that important progress with this problem will be made in the next few years.

551.508.25 : 551.594.25

The possibilities of multilayer coatings for auroral research

By A. H. JARRETT

The Observatory, University of St. Andrews

(Presented by Mr. J. Paton)

Summary

Although the examination of the spectra of aurorae provides a method of estimating temperature at auroral levels, the low dispersion of auroral spectrographs hitherto used and the long exposures required lead to uncertain results.

During the past five years it has been established that multilayer coatings are generally more efficient than their silver counterparts for interferometric work. This increase in efficiency is effected by a reduction in the absorption from, for example, 3 per cent for a silver film of 94 per cent reflectivity to 1 per cent for a multilayer coating reflecting 94 per cent of the incident light. Consequently the transparency of a Fabry-Perot interferometer can be considerably increased by the use of multilayer coatings, so reducing the exposure-time for recording fringes from an extended source.

Several trials on auroral glows have shown that it is possible to record Fabry-Perot fringes from the 5577 Å radiation with exposures of as little as 20-min duration, using simply a multilayer interferometer placed in front of an f/2 camera.

These investigations are being continued with the intention of deriving temperature distributions from measurements of the fringe halfwidths, and line-of-sight motions of the aurorae from displacements of the fringes.

Further applications lie in the use of narrow bandwidth ($\simeq$ 20Å at 5500Å and 60 per cent transmission) all-dielectric type interference-filters, particularly for investigating the form of a display in different radiations (e.g. H_α, H_β); these filters by relative enhancement of the auroral radiation will make observations possible during moonlight.

551.593.653

Noctilucent clouds

By J. PATON
University of Edinburgh

Summary

Brilliant displays of the rare noctilucent clouds have recently been seen from central Scotland, nd their height, position and speed have been determined from parallactic photographs. Obser- ations of these clouds provide information concerning the physical state of the upper atmosphere t the height of about 80 km at which they invariably occur. Besides yielding direct measurements f wind, they indicate the existence of a temperature minimum at 80 km. Wind speeds of the rder of 50 m/sec have been measured; Störmer has recorded speeds of more than 100 m/sec, nd Jesse of about 300 m/sec. On every occasion when the clouds have been observed from cotland, this movement has been from the east-north-east. But, since they are made visible y the sunlight scattered by the material of which they are composed, they are observed only n nights when the sun remains close below the horizon – that is, during the summer months n middle latitudes. Presumably they may be present at other times and in other latitudes, but hey will not remain sunlit long enough to be observed. Their nature is still in doubt, though heir tendency to appear at times of comet and meteor occurrences strongly supports the belief hat they consist of meteoric dust. E. G. Bowen has recently pointed out that they appear ither on or very near the dates of the daylight meteor streams discovered at Jodrell Bank, but Scottish observations show no such tendency. On one occasion, a portion of the clouds was bserved to change colour from vivid blue to white, suggesting growth of the cloud particles by condensation. When opportunity offers, the new spectrograph at St. Andrews will be employed o investigate such changes in the spectrum. Finally, slides were shown illustrating a remark- bly sudden change from their characteristically stable structure to quite chaotic shapes that ccurred in the early morning of 25 July 1950. Noctilucent clouds have been observed on wenty-two nights since 1939; but this was the only occasion when turbulence was seen. It may e significant that it was also the only occasion when aurora occurred simultaneously with the noctilucent clouds.

Symposium on Meteorology and Scottish Agriculture

(Held on 17 July 1954)

Climate, host and parasite in crop disease

By JOHN GRAINGER

Department of Plant Pathology, West of Scotland Agricutural College, Auchincruive, Ayr

(See p. 80 of this issue)

551.586 : 633.

Climate and the yield of cereal crops

By JOHN GRAINGER, J. L. SNEDDON, E. de C. CHISHOLM and A. HASTIE

Department of Plant Pathology, West of Scotland Agricultural College, Auchincruive, Ayr

EXTENDED SUMMARY

A series of randomised, replicated plots of 12 varieties of oats, 12 of wheat, 6 of beans and 5 of barley was established at the Department of Plant Pathology in 1945. They are, in fact, ' disease phenology plots,' but the amounts of disease on the cereals have been so minute that correlations between yield and climate are valid. Climate is, moreover, observed at a meteorological station actually within the plots. Temperatures are those recorded in a screen at 4 ft above ground level, as in normal meteorological practice.

Correlation coefficients have been prepared for comparisons of the average yield from all varieties of a crop, with monthly mean values of maximum air temperature, average air temperature, hours of bright sunshine, and rainfall, for all months during which the crops are actively growing. Table 1 gives the significant and near-significant results with yields of grain and the agriculturally useful oat straw, from a total of 206 correlations of results over 7, 8 or 9 years. Straw of barley, beans and wheat is of limited use in agriculture, but higher yields of barley

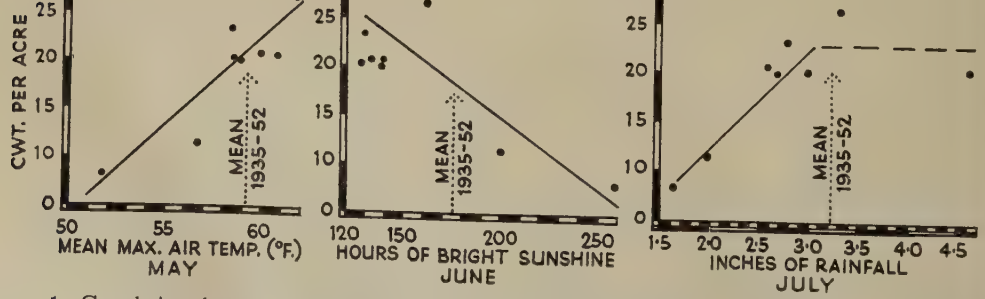

Figure 1. Correlations between the yield of bean grain and monthly means of climatic factors; see Table 1 for values of the correlation coefficients.

straw are related with lower maximum and lower mean air temperature in June. Bean straw yields higher with higher maximum air temperature in May and lower bright sunshine in June. Similar conditions in the same months are also related with higher yields of wheat straw, with which higher rainfall in July is further correlated.

Fig. 1 gives typical correlation diagrams for bean grain, and there is also a suggestion that increased rainfall in July beyond a certain value does not necessarily increase the yield.

It is important, in results of this kind, to know the average climatic conditions to which they are related, and both Table 1 and Fig. 1 give mean values for the period 1935 to 1952. The results are valid for Auchincruive, and advisory experience suggests that they apply generally to the west of Scotland. Other areas with different mean values could be expected to have different correlations.

TABLE 1. SIGNIFICANT AND NEAR-SIGNIFICANT CORRELATIONS OBTAINED BETWEEN CLIMATE AND YIELD

Higher yields are correlated with :	Month	Value of r (correlation coefficient)	Range of P	Value of r required for significance at 5% level	Long-term mean (1935-1952) favours :	Values of long-term (1935-1952) means
Barley grain						
Lower max. air temperature	June	− 0·67	0·1	0·755	Lower yields	63·4°F
Higher rainfall	June	+ 0·67	0·1	0·755	Moderate yields	2·54 in.
Bean grain						
Higher max. air temperature	May	+ 0·87	0·01-0·001	0·707	Higher yields	59·1°F
Lower bright sunshine	June	− 0·86	0·01-0·001	0·707	Moderate yields	175·4 hr
Higher rainfall	July	+ 0·68	0·1 -0·05	0·707	Higher yields	3·22 in.
Oat grain						
Lower mean air temperature	July	− 0·70	0·1 -0·05	0·755	Higher yields	58·7°F
Oat straw						
Lower max. air temperature	June	− 0·85	0·02-0·01	0·755	Moderate yields	63·4°F
Lower mean air temperature	June	− 0·70	0·1 -0·05	0·755	Lower yields	56·1°F
Wheat grain						
Lower bright sunshine	Mar.*	− 0·606	0·1 -0·05	0·666	Lower yields	102·0 hr
Lower mean air temperature	Apr.	− 0·77†	0·05-0·02	0·755	Lower yields	46·1°F
Higher rainfall	July	+ 0·80†	0·05-0·02	0·753	Moderate yields	3·22 in.

* Winter varieties only are growing in March

† These values were − 0·85 for temperature and + 0·83 for rainfall when yields were corrected for small amounts of yellow rust (*Puccinia glumarum*) on the crop in 1946 and 1948.

Among the correlations so far found, lower temperatures are more commonly related with increased yields than higher ones. Higher yield is invariably correlated with higher rainfall, but with lower bright sunshine. Living organisms have a temperature of optimum activity and it is possible that the present results with temperature merely reflect a relation to the optima for each cereal at particular times. The effect of rainfall is also understandable, but the influence of lower sunshine is less obvious. It might arise meteorologically, say, through its possible effect on temperature. It might also arise physiologically, since some plants in bright sunlight often photosynthesize so much carbohydrate that it cannot be translocated, and food manufacture stops. The process would be continuous at a lower light intensity. It could then result in the production of more carbohydrate – and a higher yield – than in bright sunshine.

Preliminary indications suggest that the aggregate yield of barley, beans, oats and wheat is, in any year, stimulated by lower air temperature, lower sunshine and higher rainfall in June and July. These relations are similar to those for the individual crops.

Practical control over temperature and sunshine is limited on an agricultural scale, but the farmer already has several methods of conserving soil moisture, which depends mostly on rainfall. A practical value of the present results lies in the precision they bring in suggesting that such conservation is most necessary at particular periods for each crop in a given area.

Discussion

Dr. O. G. Sutton (*President*) : Would the authors state at what height ' air temperature ' was measured ? It is important to recognize that the ' air temperature ' of normal climatology refers to the level at which human beings and the larger animals breathe, whereas plants, in their early stages, live in a climate which may be very different. In work of this type it is essential to state quite clearly exactly where and how the thermometers are exposed.

Mr. C. A. Halstead : I suggest to Mr. Sneddon that in connection with the consistent negative correlation between crop yields and bright sunshine at Auchincruive it should be remembered that days of bright sunshine are associated with radiation nights. Thus the correlations may be explained better as the results of frost damage (including also probably cold-air drainage) than complications of photosynthesis.

Dr. H. L. Penman : I have no questions to ask but would like to make some comments on these papers from Auchincruive. There are two aspects of the work. First, there is the attempt to predict the onset of a biological event from weather data. If this can be done successfully then it is immaterial whether the reasons for success are known. Second, there is the attempt to fit crop-growth and weather into a formula. The history of such attempts is characterized by a record of unbroken failure – we have the breakdown of the Crop-weather Scheme as a reminder of a large-scale failure – and to me it seems clear that we shall not find out *how* plants selectively integrate the effect of weather until we know *why* the weather affects plant growth and development. Progress in answering ' why ? ' questions must be slow, but at least it will be positive, and I hope the Auchincruive team will re-direct some of their effort to this end.

Mrs. Helen Goldie (*partly communicated*) : I noticed a correlation coefficient of — 0·99 between the grain yield of beans and the number of hours of bright sunshine in June. Is not a value of 0·99 so high as to be suspect ?

A glance at the full table of Dr. Grainger's correlations between crop yields and meteorological elements suggests that the number of significant correlations is less than that indicated in the summary of results presented during the meeting and a value of 0·99 now seems less remarkable.

For bean yields alone (grain and straw), 32 correlations were computed, the number of pairs in each computation being generally 8. Suppose that each pair of elements correlated are totally unrelated, then one would expect one of the 32 correlation coefficients to be of magnitude 0·77 (about) or greater. Actually three are of this order : + 0·78, + 0·87 and — 0·99. The magnitude required for 5 per cent (i.e. one-in-twenty) significance is that which would be obtained between two unrelated sets of observations only once in 32 times 20 computations of correlation. For sets of 8 observations the magnitude is about 0·99. Thus, of all the 32 coefficients computed for bean yields, the value — 0·99 is the only one which is significant at even the 5 per cent level. This correlation between high yield of grain and few hours of sunshine is probably real; but it would be advisable to check this with much longer series of observations – unless physical reasoning suggests the same relationship.

Mr. R. Cranna : Mr. Sneddon mentioned mean temperature in some instances, and mean maximum temperature in others. No mention was made of accumulated temperature; does this mean that no relationship was found, or that the relationships were not examined ?

Mr. E. P. Jeffree : I remember that R. H. Hooker carried out similar work, firstly for 21 years, and later for 35 years, for crops in eastern England, and that this work was taken up later by Dr. Geddes for NE. Scotland. It would be interesting to compare the several results particularly from the point of view of variations in different areas, which I think were sometimes evident between the results for England and Scotland.

Dr. J. Grainger (*in reply on behalf of the authors*) : In reply to Dr. Sutton, air temperature was measured by mercury thermometers at 4 ft above ground level, within a louvre screen. We do indeed make readings of various climatic factors within the crop for special purposes, but find that the air temperature of normal climatology has considerable use in the prima facie explanation of biological happenings.

We have previously had Mr. Halstead's suggestion in mind, but believe it does not apply to the present results. The air drainage from the gently-sloping disease phenology plots at Auchincruive is very good, much of it going straight down to the Ayr valley about 100 ft below, and the rest going into a tributary valley. Of actual damage, ' clipped-leaf ' and ' blind ears ' might possibly be caused by frost. The first has not occurred to any extent, and the maximum effect upon yield of the second has scarcely exceeded 1 per cent in our investigations.

I am much in sympathy with the remarks of Dr. Penman, that it is important to understand why weather has a particular effect on crops. We have, for example, now found why we must have a ' zero time ' in potato-blight forecasting. Also the *Phenological Reports* have been discontinued, but I was able to obtain from them and the *Monthly Weather Reports* some most interesting correlations of weather with plant flowering (*Ann. Appl. Biol.*, **26**, p. 684, 1939) following an assessment of the times of initial flower formation. Would these two examples show that we have the detailed aspect well in mind, though, indeed, progress is inevitably slow ?

I very much regret that the value of — 0·99 mentioned by Mrs. Goldie is an error of transcription which, owing to my absence, I was unable to correct before the meeting. The correct value is — 0·86. Mrs. Goldie's first impression should, I think, be the substantive one, for her later communication must alarm biological users of statistics. If I interpret it correctly, it appears that published tables of values of the correlation coefficient for significance at the 5 per cent level do not give sufficiently high levels in practice. The value of the correlation coefficient for significance at the 5 per cent level from 8 pairs of observations is given in Fisher and Yates as ·7067 ($n = 6$). Our own observations are being continued.

In reply to Mr. Cranna, relationships with accumulated temperature have not yet been examined. I hope to investigate this and other factors at some future time.

There must, I think, inevitably be different correlations of climate and yield in different areas, and Mr. Jeffree has raised an important matter. Correlation with other centres would be, perhaps, more potent when we have a longer series of observations at Auchincruive.

Microclimatological profiles in grass cover in relation to biological problems

By F. L. WATERHOUSE

Queen's College, Dundee, University of St. Andrews

(See p. 63 of this issue)

Soil temperatures in an afforested area in Aberdeenshire

By J. R. H. COUTTS

Department of Soil Science, University of Aberdeen

(See p. 72 of this issue)

Discussion (taken jointly) on the papers by F. L. Waterhouse and J. R. H. Coutts

Dr. O. G. Sutton (*President*) : I was very interested in Dr. Waterhouse's paper because it gives data for the actual conditions in long grass, which meteorologists generally shun, because they are studying the basic problems of exchange near the ground. What Dr. Waterhouse has shown us is a glimpse of the climatology of Lilliput, if by Lilliput you understand the sort of world in which the insect lives. What a lucky creature an insect is – it can go from a cool humid climate (like Scotland) to a Mediterranean climate merely by climbing a blade of grass ! This underlines the point I made earlier, that it is extremely difficult to define a microclimate – all that we can say is that it may be utterly unlike the climate at 4 ft above the ground.

As regards Mr. Coutts's account of soil conditions, I would like to draw his attention to the fact that it is possible to calculate the conductivity of soil directly by a simple planimeter measurement of the heat gained by the soil from successive profiles, coupled with the definition of the conductivity in terms of the flux of heat. I think this may be preferable to the classical method, which assumes that the temperature rise at the surface follows a sinusoidal law.

Dr. W. C. Swinbank : I understood Dr. Waterhouse to say that the instruments used to provide the crop profiles he showed us were aspirated. This would distort the profiles, more or less, according to the force of aspiration used. Will he tell us what was the speed of ventilation ? It is, of course, possible to make both thermometric and psychrometric elements so small that aspiration is not necessary.

Mr. J. L. Monteith : I should like to ask Dr. Waterhouse whether he has considered making a temperature-sensitive element with roughly the same dimensions and thermal properties as one of his caterpillars in the hope of obtaining a direct indication of the body temperature of a genuine grub. At present, the basic measurement in this work is an accurate air temperature from which a body temperature can be inferred, if the wind velocity and radiation intensity are also known. Three measurements are therefore needed to determine one quantity. A good caterpillar-thermometer might do the whole job at once !

Dr. F. L. Waterhouse (*in reply*) : I find Dr. Sutton's comparison of insects with Lilliputians extremely stimulating and I am relieved to be able to share with him my difficulties in defining a microclimate. The dimensions of the latter appear to vary with the problem on hand. When Gulliver is an entomologist the conditions at leaf surfaces often become all important. With this in mind Dr. Swinbank's question concerning ventilation speed is extremely to the point. In the case of the crop profiles mentioned the speed of ventilation used in aspirating my wet and dry thermojunctions was of the order of 5 cm/sec. Under the conditions of the experiment I would not expect the resulting distortion to be serious. I had heard of Dr. Swinbank's work with non-aspirated instruments and hope to gain further knowledge about an apparently ideal practice which should provide an answer to exacting experimental conditions.

With regard to the possibility mentioned by Mr. Monteith I have indeed given the matter some thought but find a number of objections to the idea as standard practice. Models of insects have been made by other workers but none is entirely satisfactory. Apart from the difficulty of making a model which exactly resembles an insect there is always the difficulty of standardizing such models so that different workers can compare their observations. Furthermore such models would only relate to the kind of insect copied. Finally, the information about microclimates so obtained would be limited in usefulness if it related only as suggested to the thermal properties of caterpillars, whereas the readings I seek are required to form the basis of laboratory experiments with respect to various aspects of insect biology. For such a purpose I see no substitute for a good set of meteorological observations. With regard to the question of insect-body temperature, I might add that several workers, including myself, have made this measurement directly in the field by inserting small junctions into insects.

551.556.2 : 551.584 : 63

The influence of shelter-belts on microclimate

By J. M. CABORN

Department of Forestry, University of Edinburgh

It is well known that belts of trees which obstruct the flow of the wind reduce its velocity in the air layers near the ground and produce a sheltered zone in the vicinity of the belts, particularly on their leeward side. A ' local ' or ' micro ' climate obtains in this sheltered area having characteristics quite different from those in unsheltered regions. Different structures of shelter-belts, in terms of width, height and penetrability to the wind, have distinct effects on the pattern of wind flow and, consequently, on the character of the microclimate. The nature of the microclimate can be assessed by measurement of the physical factors which it comprises, e.g., wind velocity, air temperature and humidity, evaporation, transpiration, precipitation, snow distribution, soil moisture and soil temperature, and also by biological means such as measurement of the yields of agricultural and horticultural crops grown in the sheltered area.

It has been proved conclusively that the solid or very dense shelter-belt does not give the most effective shelter and that a belt which allows a certain proportion of the wind to pass through it more or less undisturbed, as far as direction is concerned, is to be preferred.

Wind currents flow more or less parallel to the ground surface and increase in velocity with height above the ground. The ideal shelter-belt has the effect of lifting this flow smoothly over the belt, starting at a short distance before the belt and allowing the wind to resume its normal pattern with the minimum disturbance at some distance to leeward. This may be explained by the fact that a ' cushion ' of air develops in front of the shelter-belt, deflecting the major part of the air stream up its slope. The remainder of the air stream passes through the ' cushion ' and the belt, which acts as a filter rather than a stop, so that it emerges on the leeward edge at a reduced velocity but travelling in the same direction. Here it forms another ' cushion ' preventing the main flow over the top of the trees from resuming its normal flow parallel to the ground until it is some considerable distance from the shelter-belt. It also reduces turbulence to a minimum.

In the case of a dense shelter-belt which is almost completely impenetrable to the wind, practically the whole of the force of the wind is necessarily deflected upwards and over the belt; the high force above the shelter-belt, together with the absence of an air ' cushion ' on the leeward side, causes vigorous turbulence to leeward and the wind is forced down to the ground again at a comparatively short distance from the belt. A higher degree of shelter, i.e. a greater reduction in the wind speed, is found immediately to leeward of the belt as compared with the penetrable shelter-belt but its extent is less.

With the dense belt the wind velocity behind the belt may be as little as 15 to 20 per cent of that in the open but at 10 shelter-belt heights away it is already 80 per cent and after that there is little protection. In the case of the permeable belt the minimum velocity occurs further away from the leeward edge of the belt but at 15 shelter-belt heights the wind is still only registering 80 per cent of its unobstructed velocity (Nägeli 1946). Thus the area sheltered is considerably greater than with the dense belt.

Where the shelter-belt is too open the wind sweeps through comparatively unhindered and there is very little shelter behind it. Also, where the belt is open at the bottom only, cold draughts occur and these may be a source of danger to sheltering cattle and sheep. Therefore to protect the maximum area a shelter-belt requires to be not too open and not too dense, in other words moderately permeable to the wind throughout its height. Danish research has shown that a windbreak of approximately 50 per cent penetrability (i.e., with 50 per cent of its frontal area open), gives the most effective shelter (Nøkkentved and Flensborg 1938, 1940).

The extent of the sheltered area depends primarily upon the height of the shelter-belt. Generally the reduction of wind velocity starts at about 9 times the height of the belt on the windward side and extends to about 30 times the height on the leeward side but sometimes further. However, if we take a 20 per cent velocity reduction as the criterion of useful shelter under average conditions, the protected area becomes restricted to about 2 shelter-belt heights on the windward side and 15 to 20 heights on the leeward side. Thus a belt 30 ft high which is moderately penetrable to the wind will give useful shelter for 20 yd to windward and 150 to 200 yd to leeward. In considering ' useful shelter,' however, it must be borne in mind that even a very small percentage reduction of the wind speed may be sufficient to bring the wind velocity below a critical value for soil erosion, damage to plants, etc., and that the utility of a belt will depend very largely therefore on the absolute value of the free-wind velocity. The above figures apply where the wind direction is perpendicular to shelter-belts; where the wind strikes the belt obliquely the protected area is correspondingly shorter in extent.

In shape the sheltered zone forms a blunt triangle on the leeward side of the shelter-belt. Small regions of increased velocity occur at the ends of the belt owing to lateral deflection of the wind by the tree barrier (Bates 1944).

It has been shown how the degree of density and height of the belt affect the sheltered area. Width is not considered so important except in so far as it affects the permeability of the shelter-belt (Nägeli 1946). However it would appear that further investigation is required of the role played by the width of the belt in the flow of the wind. The Danes have achieved the optimum shelter screens with single-row belts or hedgerows. In Russia narrow shelter-belts of 7 or fewer rows are customary for arable fields provided that they have evenly-distributed, narrow, vertical openings (Gorshenin 1941); it has been found in practice that belts of 5 rows are as effective as those of 20 rows or more and the minimum widths laid down are 50 ft for main belts and 23 ft for lateral field belts. In America the typical shelter-belt is 132 ft wide but 90 ft belts of 10 rows are now frequently recommended (Woodruff and Zingg 1953). Studies

made of several belts in Switzerland show that the best velocity reduction curves were given by two belts, one 75 m (245 ft) and the other 17 m (55 ft) in width, although apparently of similar penetrability (Nägeli 1943 and 1946).

Measurements of wind velocity through wide shelter-belts show that the minimum occurs within the belt itself, from which it may be concluded that a wide shelter-belt ' consumes ' its own shelter to some extent. Experiments in Germany have indicated that a wide shelter-belt leads the wind parallel to its crown surface so that it comes down to the ground quickly after leaving the leeward edge (Blenk 1952). On the other hand, preliminary investigations made in Denmark behind large plantations indicate that the zone of shelter extends much farther behind a very wide plantation than behind a narrower one or a shelter-belt (Nøkkentved and Flensborg 1940). However, the smallest plantation considered was 200 m wide and its shelter was little more in extent that that of a narrow shelter-belt.

Investigations have also been made, where a series of parallel belts follow one another, to see if they have any cumulative effect on the wind speed. Results have differed somewhat but it is generally considered that, if belts are planted sufficiently close together, their zones of shelter overlap and they do have a cumulative effect so that at no point between the belts is the free-wind velocity attained.

Regarding the effects of shelter-belts on air temperatures : usually the temperatures in sheltered areas are somewhat higher during the day since the reduced wind velocity causes reduced air movement and consequently less exchange of heat between the air layers. The increase of temperature is usually no more than 1°C and does not extend far from the shelter-belt (Gorshenin 1941). During the night, when temperature inversion takes place, stratification of the air layers occurs in the sheltered area, the colder heavier layers remaining close to the ground. Under such conditions the danger of night frost is greater than in the open and particularly so near dense shelter-belts. This danger is minimised when belts allow a certain amount of wind to penetrate to the sheltered area, thus preventing the stagnation and stable stratification of the enclosed air.

Within a narrow strip on the edge of the shelter-belt, more factors come into play. During sunlight the shelter-belt may cause shading on one side, depending on its orientation, whilst a narrow strip on the opposite side may suffer from excessive radiation. At night the tree crowns will prevent heat loss to a certain extent by cutting off outgoing radiation. Also, at night, warm air will spread from the wide shelter-belt at least into the surrounding area. In this way there are marked temperature fluctuations on the shelter-belt margins (Geiger 1950).

Atmospheric humidity is influenced by wind, air temperature, transpiration and evaporation from the vegetation and by the moisture content of the surface soil. It is also dependent on the time of day and weather conditions. It has been established that shelter-belts cause a higher degree of humidity (2 to 3 per cent) both absolute and relative, in the sheltered region (Gorshenin 1941).

The influence on evaporation extends over considerable distances to leeward of the belt, its rate varying with the wind speed. The saving in moisture due to reduced evaporation has been found to be as much as 25 per cent with wind speeds of 12 to 15 mi hr^{-1} (Bodrov 1936). This is particularly important in areas of low rainfall. Reduced evaporation also means a reduction in heat loss, which is especially noticeable after a shower of rain.

Transpiration, the loss of water from the actual plant, should be mentioned separately. When a plant is disturbed by the wind it can lose water by transpiration to such an extent that the process of photosynthesis stops and thus the growth of the plant is hindered.

Soil moisture is appreciably higher in sheltered areas than in the open. Dewfalls are also higher and can frequently delay harvesting operations.

The effect on the soil temperature varies according to the season and the air temperature. Soil temperatures in sheltered areas are usually higher but the difference is usually less than 1°C (Bates 1911, Andersen 1943).

The drifting of snow behind shelter-belts is not so important in this country except in so far as protection to stock is concerned or the delay caused to arable farming by drifts near the shelter-belts. For arable fields shelter-belts should be more penetrable below to obtain an even distribution of snow over the fields instead of drifts on the borders. For sheltering cattle or sheep it is probably better to have the shelter-belt slightly more dense in the bottom so as to trap the snow on the windward side or within the belt itself.

Snow covering, besides adding to the moisture content of the soil, also protects winter crops and reduces the depth of soil freezing.

As a result of the effects which shelter-belts exert on the physical factors, reduced wind velocity, evaporation and transpiration and increased humidity, soil moisture and temperatures, yields of crops grown in sheltered areas are higher than in the open. There are numerous examples of this. Bates (1944) has put the maximum increase in the yields of cereal crops at 45 per cent at a distance of 4 to 5 heights from the shelter-belt. Generally there is a loss in yield from a narrow strip on the margins of the belt but increased yields extend to as far as 12 times its height. Russia claims increases in hay yields from 100 to 300 per cent (Ignatiev 1940). In Denmark increases from 6 to 34 per cent have been recorded in cereal and root crops (Nägeli 1941), and Andersen (1943) states that the increase in yield due to shelter is $4\frac{1}{2}$ times as great as the loss of yield due to location and effect of the belt. ' If shelter-belts take up 5 per cent of the cultivated area there is a 15 per cent gain even if one reckons on only a 20 per cent yield-increase due to shelter effect ' (Geete 1944).

Unfortunately we have no figures at present for the effect of shelter on permanent pasture other than that it produces an ' early bite,' probably due to the cumulative effect of shelter and not to the microclimatic changes directly.

Of the several further advantages which have been recorded for shelter-belts only a few can be mentioned here. They include reduction in mechanical damage to plants caused by strong winds; extension of the ripening season for soft fruits and cereal crops; reduction of exposure of grazing cattle to strong winds, resulting in increased production of beef and milk (the increased milk yield due to shelter has been put at 16 per cent); a better quality of hide and, in the case of sheep, a better quality of wool; production of fuel, etc.; control of erosion; prevention of soil blowing (not unknown in the Lothians in the spring) and enhancement of the property value. Shelter-belts on hill farms allow a longer period of grazing on the hill so that inbye fields can be used for fodder for winter.

From this evidence it should be apparent that there is a place in the rural economy for shelter-belts both on arable and hill farms.

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The climatological work of the Nature Conservancy

By F. H. W. GREEN

The Nature Conservancy, London

An extended summary of this paper is in the course of publication in *Weather*

551.551

The partition and intensity of eddy energy at the 91-metre level during unstable conditions as observed at Brookhaven National Laboratory

By R. A. McCORMICK

(Read 20 October 1954. Presented by Mr. H. Charnock. See *Q.J.*, **80**, p. 359)

DR. R. S. SCORER : The assumption that $K_H/K_m = 5$ for air with a land trajectory but is only equal to unity for a sea trajectory seems too arbitrary when not backed by a physical explanation. It seems possible that there would be much more evaporation from the surface into air with a land trajectory so that stirring motions would be produced by the buoyancy of the water vapour. This would be represented in Richardson's expression by an increase in $\Delta T/\Delta z$, and an excess of the order of a degree centigrade of the virtual temperature over the actual temperature could easily produce an effect of the right magnitude. Has Mr. McCormick considered this effect ?

It is difficult to avoid the impression from reading the paper that the author believes that the turbulence on all the scales he considers is produced by surface roughness, or is at least very closely associated with it. Though this is intelligible for eddies of period of a few seconds there seems to be no ground for believing that larger eddies are produced or controlled in this way. On the reasonable assumption that they are of thermal origin one would expect fewer large eddies in air with a sea trajectory if the convection is predominantly over land because the air has only been over the land for about 20-30 min on arriving at the observation point, and the large eddies do not have time to develop.

It is only to be expected that vertical velocities associated with large eddies should be small close to the ground; but it is not clear whether the author thinks that the large eddies nowhere have large vertical dimensions and are therefore flat, or whether he thinks he is only able to sample the bottom of them. This makes all the difference to any deductions he might make about vertical diffusion, unless he is going to measure correlations between vertical velocity and density of the diffused quantity. In the case of no mean horizontal wind I would expect greater fluctuations in horizontal than in vertical velocity close to the ground even when the upward ' diffusion ' of buoyancy was very large on account of convection.

In attempting to correlate the fluctuations with lapse rate and mean wind there is danger in assuming that the difference in temperature between the 125 m and 11 m levels is an equally good representation of the lapse rate at 125 m in all wind speeds. When the wind is light and the air unstable because of surface heating the temperature profile is very much curved in the lowest 20 m or so and the lapse rate at 125 m would be very much overestimated. In strong winds the small eddies produced by ground roughness, with periods of about 1-5 sec (i.e., outside the range of the measurements made) are so effective at transporting heat in the lowest 20 m that the lapse rate is less steep than in light winds when most of the transport is by convection of the kind that prevails when there is no mean wind. Thus in strong winds $T_{125} - T_{11}$ gives a much better estimate of the lapse rate at 91 m.

DR. R. C. SUTCLIFFE : I should like to ask whether the Richardson expression ' for the rate of increase of the mean turbulent kinetic energy ' used by the author is today accepted as a likely measure of that difficult quantity.

It is also rather surprising to the non-specialist in these matters to find that a direct relationship between the quantity of turbulent kinetic energy in a particular part of the spectrum of atmospheric motions and the ' rate of production ' is to be expected. It is rather like expecting to find that one's bank balance at any one time is directly related with one's income : I have not found it so. It would be interesting to have further explanation of the significance of the relationship which the author establishes.

DR. F. J. SCRASE : I should have thought that, for the purpose of deriving values of the eddy viscosity, it would have been more appropriate to have used the actual observations of wind profile made during the experiments than averages based on the power law.

Dr. F. Pasquill : I would like to refer to the last section of the paper, in which it is stated hat ' The values of K_m shown in Table 1 utilized the power law proposed by Deacon . . . ' This could mean that the Deacon formula alone was used in calculating K_m from wind measure- nents, or that the formula was used only to calculate du/dz, which was then combined with an ndependently evaluated shearing stress to give K_m. It is not entirely clear which of these pro- edures was followed, though from general context I would be inclined to agree with Mr. Charnock's nterpretation and assume that it was the latter. In any case I am surprised that both methods vere not used since this would have introduced some interesting consideration of the explicit alidity of the Deacon profile law and of the variation of shearing stress with height. Mr. Charnock as drawn attention to the somewhat surprising result that the eddy viscosities were apparently ndependent of the wind speed and shear. I wonder if this is a case of the effect of wind speed eing obscured by that of stability. If the effect of stability is related to something like the Richardson number, then it follows that the direct effect on the eddy viscosity of a change of vind speed could be nullified by an associated change in the effect of stability.

Prof. P. A. Sheppard : This paper is to be welcomed in my view because it places on record ssentially new results on the properties of smaller-scale atmospheric turbulence at a height f about 100 m. Its importance is certainly not to be minimised because it fails to give a satis- ying theory – what a triumph that would be – of the observational material presented. No loubt there are a sufficient number of theoretically minded people among us to make the author's ssay potentially very worth while. Here are results which they should certainly have before hem in the further development of theory.

Knowing the Brookhaven site, I am however rather worried about a possibly strong influence n the results of the local peculiarities of the site. Has the author for example satisfied himself hat the differentiation between ' over-land ' and ' over-water ' trajectories is not a differentiation f surface characteristics ? The heights of roughness ' elements' being only one order of magni- ude less than the height of observation, their horizontal size and their effective distribution weighted according to nearness to the tower) suggest that wind direction, independent of the roperties of the air stream, might have a considerable effect. This is not to imply that even t so low a height as 100 m the properties of air after 10 mi land-travel may not reflect properties f the air at a much higher level over the sea. There are no doubt considerable differences in he horizontal and vertical gradients of temperature in the air masses contrasted. Since eddies, f we must use the term, come from all levels, above, below and at the level – by ' come from ' do not mean simple advection – the author's differences for different trajectories may well e real. But if so, we would expect the difference to reside mainly in the large eddies. There vould be some evidence for this in the author's Fig. 4 if he had not arbitrarily introduced the ariable K_H/K_m factor; I do not understand the significance of this.

I welcome the author's use of the ' total energy supply,' as the sum of work done by shearing tress and buoyancy, instead of the more usual ratio of these quantities. I believe the device to e more important than it superficially appears, and this without prejudice to Dr. Sutcliffe's uery as to the validity of the expression for total energy production and to Mr. Swinbank's ecent query (*Q.J.*, **78**, p. 420, 1952) on the justification of Richardson's assumption that energy upply (as expressed by the author, or more completely, by e.g., Calder, *Q.J.*, **75**, p. 71, 1949) s channelled into small-scale motion. If the author waits to evaluate all terms in the energy quation he will indeed wait some time. Most workers will be aware of the limitations though he author might have been at more pains to point them out.

Again, the author has made one of the few essays in spectral analysis for the lower atmos- here. I doubt whether it shows much more than is suggested to the eye in examining a record f wind fluctuations, for the properties of his 'filter' are so indeterminate that one cannot draw ' true ' (whatever that word means) spectrum for the motions studied. This however is the ort of material which any theory must set itself to provide and though the theorist may have difficulty in using the author's results, the author's general attitude is to be endorsed.

Mr. H. Charnock (*in reply, partly communicated*) : I think Dr. Scorer cannot have noticed Section 4 of this paper, where the author says that he considers buoyancy effects are responsible or the differences in K_H/K_m which his results indicate. If he had the necessary observations e would doubtless be glad to test Dr. Scorer's suggestion that the buoyancy effects could be due to a water vapour as well as to a temperature excess. I agree, however, with Dr. Scorer hat from Section 5 one does get the impression that Mr. McCormick thinks surface roughness lays an important part. This is difficult to understand, especially in view of Fig. 4; indeed I ave wondered if the word ' conceivable ' in Section 5 was not meant to read ' inconceivable.' I annot say what the author thinks about the structure of large-scale eddies : I think Dr. Scorer

exaggerates the danger of using the temperature difference between 125 m and 11 m as a measure of stability.

Dr. Sutcliffe has referred to what in my opinion is the most interesting result of the paper. Richardson's expression for the rate of production of mean turbulent kinetic energy is not generally accepted but there is as yet no satisfactory observable alternative. Even if Richardson's expression is assumed it is not obvious that it should be proportional to the total eddy energy as defined. I would prefer to argue that the rate of dissipation was possibly proportional to the eddy energy and that in the steady state the rate of energy supply was too. One might try holding Dr. Sutcliffe's bank balance constant for some (unspecified) time and enquiring if his rate of spending were proportional to it. But this would be an expensive experiment to test an unjustified analogy. A bath tub with a tap and a narrow waste pipe would be cheaper.

The section on eddy viscosity, as I and Dr. Scrase and Dr. Pasquill have pointed out, is difficult to follow. Could it be that Mr. McCormick was not happy about the wind-speed at 11 m in view of local obstructions to the flow?

Professor Sheppard is worried about the site too but it seems unlikely that the angular distribution of surface characteristics would be such as to throw up so clearly the difference between overland and over-sea trajectories. Perhaps the wind direction should have been added to Table 1.

Fig. 4 does show that the effective K_H becomes progressively greater than the effective K_m as the eddy size increases. Presumably it is this fact which prompts one's doubts as to whether surface roughness has anything to do with it.

I must deny that the properties of Mr. McCormick's numerical filter are in any way indeterminate and assert that one can draw an exact Fourier spectrum for the motions it accepts and rejects. It may not have the sharp cut-off which perhaps Professor Sheppard would like but it has the practical advantage that hand calculations using it are tedious rather than unthinkable.

Mr. R. A. McCormick (*in reply, communicated*): To Dr. Scorer I would like to say that with regard to the ratio K_H/K_m, the point I wished to make was '. . . . about all one can say is that the ratio appears to be about 3 to 6 times greater for air travelling over land than over water.' The assignment of K_H/K_m equal to 5 for air with a land trajectory was hardly more significant, or arbitrary, than K_H/K_m equal to unity for air with an over-water trajectory. The effect of differential evaporation is an interesting concept which I had not considered.

I am in qualitative agreement with Dr. Scorer's assumptions relative to those portions of the turbulence spectrum which are sensitive to surface roughness and thermal convection, respectively. It also appears that early in the day when the convection activity is not fully developed, we are indeed sampling (at the 91-metre level) near the top of the eddies, but near the bottom later on in the day. Both of the above matters are discussed, and evidence presented, by Panofsky and McCormick in the October 1954 issue of the *Quarterly Journal* (p. 546).

It is difficult for me to appreciate how the turbulent fluctuations *on the scales considered in my paper* can be materially governed by dT/dz at 91 m. The average gradient over a considerable depth seems to be much more appropriate. Of course, the 125 m-11 m interval may not be the correct one, but of those available it seems to work out the most satisfactorily.

To Dr. Sutcliffe I would like to reply that the expression I used for the rate of increase of the mean turbulent kinetic energy can hardly be accepted as *the* measure of that (certainly) difficult quantity. Whether one would accept it as a 'likely measure' would depend on the degree of acceptance given to the observational evidence available.

In the range considered in my paper, the removal of eddy energy depends primarily on the viscous dissipation and the divergence of the energy flux. If both are proportional to the eddy energy, the relationship between the turbulent energy and rate of production would result during steady conditions. I believe a more accurate analogy is one which has been used by Panofsky. He likens the rate of production of turbulent energy to the rate of supply of fuel to the motor of an automobile, and the intensity of the kinetic eddy energy to the speed of the car. If fuel is supplied at a uniform rate, after the initial acceleration, the car will travel (on a level road) at a uniform speed. The magnitude of the speed will be proportional to the rate of fuel supply to the motor. And so, under steady conditions, the magnitude of the eddy energy will be proportional to the rate of production of turbulent energy. The friction forces and wind resistance operating on the car might correspond to the viscous dissipation and divergence of energy flux.

The point made by Dr. Scrase is well taken, and as a matter of fact, I did make the computations using observed wind profiles, although they were not included in my paper. The respective values were generally of the same order of magnitude, but I considered the ones

presented to be more accurate at the 91-m level because of the unknown distribution of curvature n the wind profiles.

To Dr. Pasquill I might point out that prior comparisons of stress computed by the Reynolds nethod and from the wind profile (see Deacon-Panofsky correspondence, *Q.J.*, p. 164, 1953) gave little promise that much could be gained by continuing in that direction. Mr. Charnock's nterpretation (I regret that one was necessary) was correct.

Prof. Sheppard's suggestion that the apparent water-land trajectory differentiation might rather be one of surface roughness differentiation, most certainly should not be discounted without ample evidence to the contrary. In fact, in the light of more recent data and analyses, I am inclined seriously to question the validity of the differentiation made in my paper, although I have nothing better to offer at the present time.

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Turbulence in the upper air, as shown by radar-wind and radiosonde measurements

By F. J. SCRASE

(Read 20 October 1954. See *Q.J.*, **80**, p. 369)

Prof. P. A. Sheppard : Observations of clouds and condensation trails indicate quite clearly that turbulent motions of the rough scale examined by Dr. Scrase vary in intensity very considerably from day to day. Moreover we have some physical reasons why this should be so. Could Dr. Scrase say therefore whether soundings made on other days than his sample show scatter diagrams differing markedly from his Fig. 2 or 3 ?

Dr. F. Pasquill : Whatever doubts may be felt about the absolute accuracy of the present estimates I suppose one might expect them to give a fairly correct description of the variation with height of the velocity fluctuations and hence of the amount of turbulent mixing. From Table 1 it is seen that the fluctuations apparently increase slightly with increasing height. My impression is that considerations of the diffusive separation of the rarefied gases in the upper atmosphere and measurements of the distribution of water vapour have suggested values of diffusivity in the lower stratosphere in the range 10 to 10^3 cm^2 sec^{-1}, whereas the corresponding values in the troposphere are usually given as 10^4 and above. There is clearly some discrepancy between this variation with height and that which is implied by the estimated fluctuations.

Dr. R. S. Scorer : Dr. Pasquill has discussed the diffusion that would result from eddies of the kind observed by Dr. Scrase. In the case of Dr. Frith's measurements of ' eddies ' of similar size by aircraft of the Meteorological Research Flight the fluctuations could be *largely* explained as wave-like motion which is inevitable over unlevel ground when there is a horizontal wind. If the fluctuations noted by Dr. Scrase are of a similar kind they would not produce any vertical transport of water vapour, heat, or any material constituent of the atmosphere. Before suspecting the observations on the ground that the eddies claimed to be observed would produce too great transports it is necessary to decide whether the fluctuations are necessarily due to a kind of motion that would actually produce any transport.

It is a dangerous error to assume that fluctuations of velocity or temperature are necessarily associated with diffusion.

Dr. R. C. Sutcliffe : If, as I believe, Dr. Scrase's analysis is of real importance it is worth remembering that there are available some thousands of ascents which could be analysed similarly. The analysis would be laborious but it should be worth while to pursue a good deal further.

Concerning the distinguishing of real variability from errors of observation is there not some possibility of obtaining internal evidence from the nature of the variations? In certain cases it is easy to show statistically that variability is not due to errors, random or systematic. The coherence of the sequence of observations in the ascent, showing a scale of perturbation of several thousands of feet, suggests physical reality. This could be checked by quantitative analysis.

Mr. J. S. Sawyer: In synoptic research the need is frequently felt for an estimate of the intensity of turbulence in the free atmosphere and Dr. Scrase's attempt to obtain this information is very welcome.

The apparent increase in the horizontal eddy velocities from the troposphere to the stratosphere, is intriguing and confirmation is desirable. Can it be that the greater vertical stability of the stratosphere suppresses vertical mixing, and, in the absence of such coupling between adjacent levels in the stratosphere, the horizontal eddy motions have greater opportunity to develop?

Mr. A. S. Mathewman: Kellogg in America has analysed observations on 18 smoke-puffs at heights ranging from 24 to 63 thousand feet, and he obtained the result that the root-mean-square eddy velocity increased with height (*added later*: actual values increased from 4 to 10 cm sec^{-1}).

Dr. O. G. Sutton: There are two points I should like to put to Dr. Scrase. His results, taken at their face value, indicate that, at all heights, the gustiness of the wind and the amplitude of the temperature fluctuations are of the same order of magnitude as those found near the surface. This is difficult to explain on any theory at the present time. The persistence of contrails, for example, shows that winds in the high atmosphere are not as effective as winds near the ground in breaking up clouds. I think it must be true that, for short-period fluctuations, the turbulence of the wind usually decreases as one leaves the ground, but I am not so certain about long-period oscillations.

All this makes me wonder if part of the fluctuations given here may not originate in the use of a balloon, which is no more than a bubble, subject to irregular heating and cooling as it rises. It may be that even long-period averaging would not remove the consequent irregularities. Did the ascent analysed here take place in sunshine? Also, I do not feel entirely happy about the estimate of the random error. At the longer ranges the effects of noise must be significant.

Dr. F. J. Scrase (*in reply*): I would like to emphasize that the primary purpose of the investigation was to test the practicability of the method for giving rough estimates, rather than accurate measurements, of the gustiness of the upper air, on which information was badly needed for high-altitude aviation.

With regard to Professor Sheppard's question, I have examined by this method the fluctuations in the vertical direction in only two other soundings and they showed scatter of about the same magnitude as that of Fig. 2.

I agree with Dr. Pasquill that the apparent increase of the velocity fluctuations, and the implied increase in diffusivity, with height is surprising. But as mentioned by Dr. Goody in his monograph on the stratosphere, similar increases have been reported by several other investigators using balloon observations and, as Mr. Mathewman has mentioned, by Kellogg who used smoke-puff observations.

Dr. Sutcliffe reminds us that thousands of soundings are available for similar analysis. This analysis is laborious and I hope that the autographic records which the radarsonde theodolite will provide will give us information on the fluctuations much more readily.

The reality of the variability, at any rate up to 12 km, has been confirmed by comparing simultaneous observations made with two radar sets. Both showed most of the fluctuations to the same extent. Moreover, as Dr. Sutcliffe suggests, the scale of the fluctuations of several thousands of feet suggests physical reality rather than pure statistical variation.

I agree with Dr. Scorer that we cannot be certain that the fluctuations of velocity or temperature are necessarily associated with turbulent diffusion rather than with wave-like motion. Mr. Sawyer's suggestion that the suppression of vertical mixing in the stratosphere allowed greater horizontal eddy motion to develop is interesting, but I do not recall evidence of such an effect in stable layers generally.

Dr. Sutton wonders if part of the fluctuations may not originate in the irregular heating and cooling of the balloon. This is unlikely in the ascent analysed since it was made at night. With regard to the estimates of random error of the radar measurements, the effects of noise become significant just before the extreme range is reached, but except perhaps for the last two or three readings, the signals are measurable without appreciable increase in error.

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Five flights through a thunderstorm belt

By R. F. JONES

(Read 20 October 1954. See *Q.J.*, **80**, p. 377)

Mr. W. E. H. Hollings : The general problem of the use of aircraft in the determination of gust velocities is being considered in some detail by the Meteorological Research Flight, and I would here like to comment briefly on some of the difficulties arising from the application of aircraft accelerometers to this problem.

Firstly, there are those problems which are inherent in an accelerometer and those which arise from interaction between it and the aircraft. When an accelerometer is subject to transient acceleration pulses it cannot record them faithfully in complete detail. This is a fundamental mechanical problem of the accelerometer and can be overcome to some extent by correct adjustment of its characteristics. It can be shown theoretically that if the damping is 0·6-0·7 of the critical damping, fairly accurate representations will be given in most cases. Zero damping or critical value damping will give results wildly in error. Another requirement for an accelerometer is that the natural period of the instrument should be less than 0·3 (approx.) times the duration of the transient pulse. This is a much more difficult problem as neither the average size and duration of gusts nor their distribution is known to any degree of certainty. If the duration range is large, this aspect of our work becomes almost intractable. The next series of problems arises when we consider an accelerometer fitted into an aircraft. For all parts of an aircraft, and in particular the wings, have their own natural frequencies of vibration. Unless care is taken, the accelerometer trace will contain in large part a record of the excitation of these frequencies, together with their overtones and combinations. It is generally considered that an ideal aircraft accelerometer should have 100 per cent response up to 2·5 times the natural frequency of the wings and zero response for all higher frequencies. This stringent condition is almost impossible to attain with simple, single-mass instruments, but double-mass accelerometers can be designed to have these characteristics. The existence of these natural frequencies, together with the fact that an aircraft rolls, pitches and yaws in a gust, make it imperative to fit the aircraft with several accelerometers, including one at the centre of gravity, one in the nose, one in the tail, one at each wing tip and others at the nodal points of the wings and fuselage. The interpretation of these records becomes progressively more tedious.

The next major consideration is the response of a given aircraft to a given gust. As gusts are considered to occur suddenly and to be of relatively short duration, this involves the difficult study of the motion of a body in unsteady flow and so far as I am aware, no completely satisfactory theory, which at the same time is suitable for practical work, has been propounded; indeed it is unlikely that this is possible. When an aircraft wing experiences a sudden change in angle of attack, such as it does when it enters a gust, the flow over the wings becomes unsteady, and there is a considerable time lag before the flow is steady again. This makes the normal aerodynamic forces temporarily non-operative. Further, since an aircraft suffers vertical motion in a gust any acceleration recorded on the aircraft, e.g. at the centre of gravity, will be alleviated to some extent, sometimes positively, sometimes negatively.

It is convenient to deal primarily with sharp-edged gusts as other types can be dealt with by the correct combination of a number of sharp-edged gusts. We can then attack the problem with the aid of two ' unsteady lift ' functions. The Wagner function deals with the growth of lift due to a sudden change in angle of incidence, and the Küssner function deals with the change in lift experienced owing to an aerofoil penetrating a sharp-edged gust; the latter takes into account vertical motion of the aircraft. It is possible, by utilising these functions, to set up an equation of motion for the aircraft, which, if not ideally accurate, gives a very fair approximation to the actual motion. Uusally, distance travelled after the initial disturbance is plotted against the force function. The force function is defined to be the ratio of the actual force acting on the wing to the force which would be acting if the Wagner effect and the vertical motion of the aircraft were absent. It is found that the major factors involved are : (1) the ratio of inertial

forces to aerodynamic forces of the aircraft, called the mass parameter, (2) the extent and shape of the gust and (3) the aspect ratio of the wing. Now the integrated accelerometer trace will give the vertical motion of the aircraft, not that of the air, and it is therefore necessary to know how long an aircraft will require to take up a given percentage of the gust. This time can be obtained from the force function curve and it is interesting to note that, on preliminary calculations I have made for the Meteor 7, it appears that the aircraft requires a time of the order of 2-3 sec to take up 90 per cent of an isolated sharp-edged gust.

When an aircraft meets a series of arbitrary gusts the problem becomes much more difficult for several reasons. Firstly an aircraft acts in two ways as a filter for gusts. It will almost completely ignore small gusts with high frequency and on the other hand will follow fairly faithfully very long gusts of small frequency, such as lee waves. Also, resonant frequencies in the structure can be excited if these have a certain relationship to the gust frequencies. Another difficulty is that the force function curve becomes very complicated if certain combinations of gusts are encountered. If the aircraft meets a double triangular gust, with one peak positive and the other negative, it will react to the second peak in a manner very different from what it would if the first peak had been absent. The relationship between the accelerometer trace and the gust profile in the case of a series of differently shaped gusts is thus very complex.

By assuming that the forward speed of the aircraft is unchanged and and that the gust is vertical, we can, on differentiating the fundamental lift equation, obtain a formula which relates acceleration increment with instantaneous gust velocity. This is of the form $u = 2W\Delta n/S\rho_0 V\partial C_2/\partial\alpha$ where u is the gust velocity, W is the weight of the aircraft, Δn is the acceleration increment, S is the wing area, ρ_0 is the sea-level density of air, V is the forward speed of the aircraft and $\partial C_2/\partial\alpha$ is an aerodynamic constant of the aircraft considered. This formula may be applied to extreme points of the accelerometer trace to obtain some idea of the gust profile. Unfortunately, in order to take into account the aerodynamic difficulties mentioned above, it is necessary to insert into this formula a constant which is a function of the shape of the gusts. The shape of a gust is not obvious from the accelerometer trace.

Even refined treatments of the problem make many assumptions which are well known to be incorrect and thus affect the accuracy of any results obtained. The main assumptions are (1) the forward speed of the aircraft is unchanged, (2) no rolling, pitching or yawing takes place, (3) the gust is uniform across the span of the wing. The exact effect of these assumptions is difficult to estimate and further work is needed before definite estimates of accuracy, when these factors are neglected, can be given. Further difficulties arise with regard to the flexibility of the wings, and of the fuselage, interaction between the fuselage and the wings, the architecture of the wings, and tailplane characteristics.

The last major consideration is the fact that a pilot is controlling the aircraft. Any natural motion of the aircraft will usually tend to be damped out to some extent by his action on the controls. Ideally, we ask the pilot to fly at constant power setting and constant attitude, but in violently turbulent conditions this is not physically or psychologically possible.

We see that in order to make a fairly accurate estimate of gust profiles from accelerometer traces, it is necessary to have continuous records of acceleration, horizontal velocity, altitude, angle of attack and preferably other items characterizing the aircraft's motion, a record of all pilot's control motions and an exhaustive knowledge of the aerodynamic and mechanical constants of the aeroplane.

In conclusion, I would like to say that as a consequence of all these difficulties, an aircraft accelerometer is not an ideal instrument for gust research. Provided the initial instrumental difficulties are overcome, the general picture of shape and distribution of gusts will be reasonably accurate, but the magnitude of gust velocities is sometimes likely to be in error by up to 50 per cent. An aircraft fitted with several ideal accelerometers, and having a small mass parameter, can be used to give a fair approximation of gust velocities, but in view of the complex nature of the problem, I think that a new approach to the problem is really needed, in which an aircraft itself is not a primary measuring instrument.

Dr. Joanne S. Malkus: It is gratifying to learn about the work that groups in Great Britain are doing in cumulus-cloud investigations by means of aircraft, and it is hoped that by comparisons of methods and results on the two sides of the ocean, each of the several groups may profit.

It is perhaps worth while, in evaluating the accuracy, meaning, and significance of the vertical draught measurements presented here to compare the method by which they were obtained with the method used by our group at the Woods Hole Oceanographic Institution, U.S.A., involving a PBY-6A (Catalina) aircraft.

There are essentially two ways in which vertical draught velocities may be measured from an aircraft in flight. The first involves obtaining an expression for the variation of the lift force acting upon the wings in terms of the vertical gust or draught velocities and other quantities which can be measured, such as airspeed, pitch angle, vertical acceleration, aircraft mass, wing area, lift coefficient as a function of angle of attack, etc. This is the method employed by us at Woods Hole. The second approach consists in finding the vertical motion of the aircraft by means of differentiating an altimeter record or integrating an accelerometer record, and applying to that result an experimental correction for the aircraft's rising or sinking speed relative to the air. The latter correction is, in this method, determined by a flight calibration. The second method clearly sounds simpler but, in our view, the simplicity is only apparent.

In using an accelerometer in either of these ways for meteorological purposes, certain preliminary tests are essential, to ascertain that the accelerations measured within the aircraft fuselage are in a known proportion to the actual gust loads imposed upon the wings and not distorted in a complicated or unknown manner by spurious responses of the aircraft itself and by relative motions between wings and fuselage. Various tests may be made to determine this : in the case of our PBY, strain gauges were mounted on the wing struts and their readings in all types of flight compared with those of the fuselage-mounted accelerometers. Wing whip, if present, will show up as spurious accelerations, and must be measured and corrected for. In the case of the PBY, wing whip was of sufficiently high frequency and small amplitude to be neglected. Its neglect is far more dubious in an aircraft without strut-mounted wings. The best test in practice, however, that all these conditions are met is that the once-integrated accelerometer record agrees closely with the once differentiated record of a simultaneously recording sensitive altimeter. In the case of the PBY this comparison works out nearly perfectly after the accelerometer has been placed in a temperature-controlled box so that its zero holds accurately. In the case of the results just presented, the disagreement between these two records, and more especially the fact that the zero correction necessary to make the accelerometer integration agree with the altimeter wanders from positive to negative within cloud but very little outside, indicates the possibility that the cloud gusts are exciting fictitious responses on the part of the aircraft or that they are resonating to some degree with the natural frequency of the wing whip. It would therefore not seem safe to rely upon the details of the results until these points have been cleared up.

A perhaps more serious source of error in detailed interpretation of the draught structures presented lies, however, in the assumption that constant aircraft attitude is maintained within cumuli. We at Woods Hole attempted use of this assumption for a while and later introduced a recording gyro to provide a record of pitch at all times. In less than 20 per cent of the cumulus flights made was it possible for practised pilots to vary attitude by less than $\pm 2°$. For obtaining the average ascent rate across an entire draught (as we were interested in doing up until recently) it may be shown that the errors arising from these departures are small indeed but, even if draught values are wanted every $\frac{1}{5}$ sec, the errors for each degree pitch variation will be about 1 m/sec, while $\frac{1}{10}$ sec readings are not feasible. Contrast this with errors arising from an assumption of constant airspeed, which for our set-up will not exceed 25 cm/sec for each 1 m/sec airspeed deviation. It is to be remembered that our aircraft weighs 36,000 pounds and being so heavy and relatively unresponsive will change its attitude far less upon encountering turbulent gusts than would a smaller and lighter craft.

It can therefore be anticipated, and may indeed be demonstrated, that the calibration used in the paper presented for getting the aircraft's sinking speed is only accurate for level attitude and probably extremely inaccurate when the attitude departs, as it must in cloudy air. In other words, the sinking speed is not a single-valued function of airspeed and power setting, but may differ widely when the pitch angle is changed, though the power setting and airpseed are held constant. Nor has it proved in the case of our aircraft that airspeed fluctuations are accurate indicators of pitch change; first the pitch change must be maintained some time before showing up in airspeed records, and airspeed may change at constant pitch owing to cloud turbulence. Thus to rely on the details of the records presented, it would be necessary to go from one calibration curve to a whole family, made for each degree variation in pitch angle and to record pitch continuously on each flight. The method becomes therefore at least as complicated as the one employed by us using lift variations. Furthermore, we have found in order to get really accurate draught structure, one must correct for the degree of gust penetration by the wing, and also must demonstrate that departures from steady flow around the airfoil are negligible (truer for slow, heavy, unresponsive aircraft).

I should like to congratulate the writer, however, despite any reservations raised in these comments, upon his perseverance and fortitude in carrying out these valuable measurements and on his courage in flying into a cumulonimbus exhibiting lightning, which we at Woods Hole have been so far too timid to dare. And I should like to conclude by mentioning (despite the pessimistic views of the previous speaker and others who without making specific tests have nevertheless somehow come out with a figure of 50 per cent minimum errors for this method) that independent checks upon the PBY results show the error to be more like 15 per cent, or less, for one-fifth second draught readings. Several lines of evidence are now available to show that with a suitable aircraft and sufficient testing, study, and perseverance, this type of measuring technique can be made to give results which are meteorologically useful.

MR. R. F. JONES (*in reply, communicated*) : I was not unaware of the difficulties involved in interpreting aircraft accelerometer records in terms of gust velocities. As Dr. Malkus and Mr. Hollings have pointed out the problem is very complex, nor is there universal agreement on how to derive true air velocities from aircraft-instrument readings, however many instruments and readings are used. The work of Dr. Malkus and her group on cumulus clouds has been pioneer work in this respect and is of great importance in determining the factors associated with the initial growth of such clouds. But I doubt, quite apart from the great labour involved, whether a similar detailed approach would be of such value in the large mature cumulonimbus clouds. In my paper I have sought to show what an aircraft will experience in traversing a thunderstorm cloud and to relate that experience to the major up and down currents within the cloud. I do not expect to show the fine detailed structure which, in any case, is continually changing and am satisfied if the magnitude of up and down currents deduced are accurate to the 15 per cent Dr. Malkus quotes.

These flights and many others were primarily designed to gain information on the aircraft accelerations encountered within cloud and their relationship to the radar echo from the cloud, but it seemed worth while to indicate, by using this selection of five flights, how the very simple approach I have used may still give useful information on thunderstorm structure.

Finally, on behalf of the pilot and observer, I must thank Dr. Malkus for her kind references to fortitude and courage which, since I was *not* in the aircraft on any occasion, I can most heartily endorse.

Proceedings at meetings of the Society

Ordinary Meeting, 20 October 1954

The following candidates were declared to have been elected by the Council during the summer recess :

Fellows

Chande, Manohar, 17 Kohinoor Road, Bombay 14, India.
Chin, Ping Chuen, B.Sc., D.I.C., Royal Observatory, Hong Kong.
Damms, Jack, 107 Church Road, Skegness, Lincs.
Filmer, John Jesse, Links Cottage, 177 Island Wall, Whitstable, Kent.
Heastie, Harry, M.Sc., 101 Old Park Ridings, Grange Park, London, N.21.
Khan, Jamil Ahmad, B.Sc., D-10, Karachi Airport, Karachi No. 11, Pakistan.
Lighthill, Professor Michael James, F.R.S., The University, Manchester 13.
Fox, Michael Matthews McDermott, Box 5163, Nairobi, Kenya.
Moore, John Gilbert, B.Sc., 105 Heathwood Gardens, Charlton, London, S.E.7.
Morton, Donald Edward Joseph, 22 Boltons Court, 220 Old Brompton Road, London, S.W.5.
Nelson, Ronald Ian, 10 Randolph Crescent, Edinburgh 3.
Noble, John W., B.Sc., 60 St. Andrew Street, Galashiels, Selkirkshire (*transfer from Student Membership*).
Owen, Paul Robert, B.Sc., A.F.R.Ae.S., 36 Oak Road, Withington, Manchester 20.
Parry, Maurice, M.Sc., 15 Cawsam Gardens, Caversham, Reading, Berks.
Shamshad, Khan M., B.Sc., Karachi Airport, Pakistan.
Snodgrass, Catherine Park, M.A., Ph.D., 16 Camus Avenue, Edinburgh.
Webb, Charles George, B.Sc., Head of the Department of Physics, University of Malaya, Singapore 10.

Foreign Members

Lenhard, Capt. Robert W. Jr., M.A., United States Air Force Headquarters, 28th Weather Squadron, Bushey Park, Teddington, Middlesex.
Merewether, Arthur Francis, Ph.B., M.S., C.B.E., Superintendent of Weather Services, American Airlines, La Guardia Field, N.Y., U.S.A.
Shuman, Frederick Gale, Sc.D., 3200 Terrace Drive, Apartment B, Washington 23, D.C., U.S.A.

Student Members

Jackson, Charles Ian, 782 King Lane, Allwoodley, Leeds, Yorks.
Nunn, Martin William, 3 St. Stephen Road, Barnehurst, Kent.

The following candidates were then declared to have been elected that day by the Council :

Fellows

Adams, John M., B.Sc., 3 Church Road, Leatherhead, Surrey (*transfer from Student Membership*).
Bovill, Robert Clive, 49 Perry Avenue, Birmingham 22b.
Burrows, Instr. Lieut. Albert Richard Godfrey, M.A., R.N., 14 Stamford Road, West Southbourne, Bournemouth, Hants.
Cheek, Neville, 66 Thorndon Gardens, Ewell, Surrey.
Cowie, Harold John, 3 Ennerdale Drive, Kingsbury, London, N.W.9 (*reinstated*; previously elected 1939, resigned 1948).
Earles, Flg. Off. Brian Peter, c/o Royal Air Force Club, 128 Piccadilly, London, W.1.
France, Margaret Norris, B.A., St. Martin's Paddock, Canterbury, Kent.
Grieve, Geoffrey Relf, 20 Wayville Road, Dartford, Kent.
Heffernan, Kenneth James, 181 Fullers Road, Chatswood, Sydney, Australia.
Jones, Instr. Lieut. David Elwyn, M.Sc., D.I.C., A.R.C.S., R.N., 187 Vicarage Road, Leyton, London, E.10 (*transfer from Student Membership*).
Ormiston, James Robertson, B.A., Trood House, Alphington, Devon.
Rogers, Michael H., B.Sc., Ph.D., University of Illinois Observatory, Urbana, Illinois, U.S.A.
Sayer, Herbert John, Field Officer, Desert Locust Survey, Box 231, Asmara, Eritrea.

Foreign Member

Constantacopoulos, Constantine, D.I.C., 73 Oxford Gardens, London, W.10.

Student Members

Adams, James A., Post Office House, Balnamore, Ballymoney, Co. Antrim, Northern Ireland.
Baalham, Gillian Louise, Cinderfield, Meath Green Lane, Horley, Surrey.
Baker, Georgina, 4 Mitcham Grange, Taplow, Nr. Maidenhead, Berks.
Basden, Julia, 4 Furnival Avenue, Slough, Bucks.
Boal, Frederick Wilgar, 19 My Lady's Mile, Holywood, Belfast, Northern Ireland.
Collison, John, 11 Holly Park Grove, Bradford 7, Yorks.
Douglas, Barbara, 134 Front Street, Sowerby, Thirsk, Yorks.
Feaver, Russell John, 75 Sandilands Road, Walton-on-the-hill, Tadworth, Surrey.
Grace, Wiliam Arthur George, 23 Belle Vue Road, Andover, Hants.
Greenaway, David Stuart, 1 Railway Cottages, Twickenham (Station), Middlesex.
Ingram, Miss Glenys, 18 West Street, Earls Barton, Northants.
Jeffery, Paul, 24 Woodland Terrace, Plymouth, Devon.
Pickering, John W., 35 Kingsland Avenue, Coventry, Warwickshire.
Singleton, Frank, B.Sc., 40 Courtfield Rise, West Wickham, Kent.
Smith, Laurina R., 101 Kent Road, Grays, Essex.
Strange, Gillian, 24 Museum Road, Oxford.
Walker, Peter Henry, c/o The University Union, Burlington Street, Manchester.
Watt, James Cameron, 24 Sunnyhill Grove, Exley Head, Keighley, Yorks.

The following papers were read and discussed :

(1) ' The partition and intensity of eddy energy at the 91-metre level during unstable conditions as observed at Brookhaven National Laboratory ' by R. A. McCormick (*Q.J.*, **80**, p. 359). The paper was presented by Mr. H. Charnock.

(2) ' Turbulence in the upper air, as shown by radar-wind and radiosonde measurements ' by F. J. Scrase (*Q.J.*, **80**, p. 369).

(3) ' Five flights through a thunderstorm belt ' by R. F. Jones (*Q.J.*, **80**, p. 377).

Ordinary Meeting, 17 *November* 1954

The following candidates were declared to have been elected that day by the Council :

Fellows

Bethell, Richard George, Rock Cottage, Llandogo, Chepstow, Monmouthshire (*reinstated*; previously elected 1945, resigned 1953).
Freshwater, Philip Rowland, F.R.A.S., c/o Hambros Bank Ltd., 41 Bishopsgate, London, E.C.2.
Lackersteen, Roy Eric, 37 Church Road, Richmond, Surrey.
Laird, Alan Robertson, M.A., 225 Essex Square, Salisbury, Wilts.
Martin, John Hamilton Anthony, B.Sc., 132A Shooter's Hill Road, Blackheath, London, S.E.3.
Mills, William Howorth, 28 Main Street, Kimberley, Nottingham.
Mowat, Michael John Dennison, B.Sc., 10 Merchiston Bank Gardens, Edinburgh, 10.
Russell, Edward Lloyd, 21 Sydenham Hill, London, S.E.26.

Foreign Members

Blackadar, Alfred K., Ph.D., Department of Meteorology, The Pennsylvania State University, State College, Penna., U.S.A.
Imai, Ichiro, D.Sc., Chief of the Aerological Laboratory, Meteorological Research Institute, Mabashi, Suginami, Tokyo, Japan.

Associate Member

Briggs, Keith Douglas, Officers' Mess, R.A.F. Abu Sueir, M.E.A.F. 10.

Student Members

Andrews, John Browning, 68 Friars Street, Sudbury, Suffolk.
Hakim, Salahe, 10 Mervan Road, London, S.W.2.
Langham, Edward Joseph, 47 Vale Road, Worcester Park, Surrey.

The meeting took the form of a discussion on 'Water supply' and was opened by the following speakers :

Mr. D. Halton Thomson, formerly Engineer to the Portsmouth Water Company, who gave a general survey of the water problem in Great Britain; Dr. J. Glasspoole, Meteorological Office, who spoke on British rainfall, giving a survey of income; Mr. N. A. F. Rowntree, chairman of the Hydrology Research Panel, Institution of Water Engineers, who spoke on surface water and underground water, and Dr. H. L. Penman, Rothamsted Experimental Station, who spoke on seasonal and long-term changes in the components of the water balance of a catchment.

The meeting was then thrown open to general discussion.

The meeting was followed by an informal dinner, which was held at Ayrton Hall, Imperial College, London, and was attended by 50 members, guests and wives. Professor P. A. Sheppard, Vice-President, was in the chair and the guest of honour was Mr. W. C. Knill, past President of the Institution of Water Engineers.

Ordinary Meeting, 15 *December* 1954

The following candidates were declared to have been elected that day by the Council :

Fellows

Coleman, Henry Nii Ayi, Gold Coast Meteorological Service, Accra.
Jones, Peter Alun, B.A., 12 Leiston Road, Theberton, Leiston, Suffolk.
Krill, Albert Dennis, 4 Cross Park, Pennar, Pembroke Dock, Pembs.
MacPherson, Alan Gibson, M.A., Department of Geography, University of St. Andrews, Scotland.
Sibbons, John Louis Harry, B.A., B.Sc., Department of Geography, The University, Sheffield 10.

Foreign Members

De Vries, Dr. Daniel A., Laboratory for Physics and Meteorology, Agricultural University, Wageningen Hamelakkerlaan 4, Wageningen, Holland.
Killory, Major Martin Francis, B.Sc., 237 West Wycombe Road, High Wycombe, Bucks.
Wlassova, Zena, Standish House Sanatorium, Stonehouse, Glos.

Associate Member

Greeve, Dennis Arthur, 4 Government Road, North Claremont, Western Australia.

The following papers were read and discussed :

(*a*) 'A study of blocking in the Atlantic-European sector of the northern hemisphere' by E. J. Sumner (*Q.J.*, **80**, p. 402).

(*b*) 'Short-period variations in surface pressure and wind' by I. J. W. Pothecary (*Q.J.*, **80**, p. 395).

(*c*) 'Secular changes in the rainfall regime of SE. Australia' by E. B. Kraus (*Q.J.*, **80**, p. 591). The paper was presented by Prof. Gordon Manley.

Agriculture; The Journal of the Ministry of Agriculture, Diamond Jubilee Issue. London. H.M.S.O. Vol. LXI. No. 6. Sept. 1954. 1s.

To celebrate the Diamond Jubilee of the publication of their official journal *Agriculture*, the Ministry of Agriculture and Fisheries invited a wide range of well-known personalities in agricultural science and practice to review the development of their special interests in the 60 years which have elapsed since 1894 : the result is a compact, but comprehensive, survey, equally interesting to layman and farmer.

In that period all phases of British agriculture have experienced changing fortunes : scientifically, it was a period of continuous and rapid advance; economically and socially, it was one with great fluctuations. Apart from an account by L. P. Smith of some of the main features of the weather of that period – illustrating amongst other things the great value of quite elementary methods of analysis – there is a curious lack of explicit reference to weather and climate. This is very strange for, as one contributor reminds us, although each member of the human family is entitled to an estate of 15 acres; of these, five are jungle, six either desert or semi-arid, and two are polar ice, leaving only two to make a contribution to the world's larder. Here, surely, is an implied challenge to the meteorologist which must not be ignored.

R.W.G.

Vision through the atmosphere. By W. E. Knowles Middleton. University of Toronto Press, 1952. Pp. xiv, 250; 120 Figs., 26 Tables. 2nd Edition. 68s.

Vision is an ambiguous word. It usually means seeing ' not with the eye but with the mind '; mystical foresight or the irruption into consciousness of a revelation of transcendent beauty; the only prophylactic against extinction. But it also means seeing with the bodily eye, what Drs. Ida Mann and N. Pirie call quite simply ' The science of seeing.' For this kind of vision, Mr. Middleton has given us a book of wide interest, provocative of constructive thought, and, occasionally, of gentle protest. It should be of outstanding benefit to the successors, in the World Meteorological Organization, of the Commission for Synoptic Weather Information and to all meteorologists concerned with the provision of information about visibility, to shipping, aviation, transport, photographic and film services. They should not be put off by Middleton's omission from this book of the climatological and synoptic chapters of its predecessor on the ground that he ' no longer feels competent to discuss such questions ' but should rather trust his well-founded belief ' that meteorologists may still read the present work with profit, if not with much self-satisfaction.'

The book contains eleven chapters whose titles provide a useful indication of the nature of the subject and the logic of the author's mind, a most valuable bibliography, a table of notation and an index of subjects but not of names of authors. This omission I found a real inconvenience. In a short introduction, the visibility problem – the determination of the distance one can see through the atmosphere or what one can see at a given distance – is outlined, with emphasis on the need for co-operation between the meteorologist, the physicist, the physiologist, and the psychologist for its solution. Middleton uses the term ' visual range ' for the distance that something can be seen. He says ' this term was introduced into English by M. G. Bennett (1930) . . . Bennett said that he used the word ' visibility ' to express the clearness with which objects stand out from their surroundings under conditions of good seeing, when all are visible; and this agrees with common usage in non-technical fields . . . ' The reference, Bennett (1930), is, according to the Bibliography, Bennett's paper in the *Q.J.* for Jan. 1930. The term, visual range, does not appear in this paper and the definition of visibility in the paper does not restrict it to occasions of good seeing. Indeed, he says ' as the obscuration increases the visibility of the object is reduced until finally it may become zero.' Moreover, in common usage on this side of the Atlantic, visibility means the condition of being visible, without restriction to occasions of good seeing; and it also means the range of vision, not merely in technical fields but in general newspaper and broadcast descriptions of weather compiled by journalists for the information of the common, and uncommon, man. The word ' visibility ' was used in the Beaufort scale of weather phenomena to denote ' great transparency or clearness of the air, rendering distant objects unusually visible.' This description was transmuted by resolution of the International Meteorological Committee in 1937 to Dr. Bergeron's greatly improved form, ' Pure air – the atmosphere has a great clearness and transparency : distant objects and their details stand out in full relief from the background with great hardness and distinctness (resembling the ' objects ' in a stereoscopic view), without any softening veil at 10 km distance or more.' Bennett used the term ' visual range ' in his 1932 paper but, in 1935, he used ' visibility ' with its ordinary meteorological meaning.

As ' Middleton ' is likely to be for many years the standard book of reference he might well clear up the apparent discrepancy in the next edition, and perhaps, setting the practical advantages of the term ' visibility ' against the natural objections, end the war of words. He does admit ' there is every indication that " the visibility " will live forever.'

The chapters after the introduction deal in sequence with Elementary photometric theory, The extinction of light in the atmosphere, The alteration of contrast by the atmosphere, The relevant properties of the eye, The visibility of objects in natural light, The visibility of light sources and of objects illuminated by artificial light, The colours of distant objects and the visibility of coloured objects, Instruments for measuring visibility, The special problems of the meteorologist, A new visual science – Conclusion. Middleton uses ' Visual range ' in these titles instead of Visibility.

In the brief chapter on photometric theory, which contains both a clear exposition of the laws of photo-

metry and tables of the concepts of radiation and illumination with explanatory notes, Middleton mentions and dismisses the nebule. When he says a nebule is really only an odd unit for the extinction coefficient, he conveys, I think, a wrong impression. The nebule is a defined quantity of obscuration in the path of light, irrespective of the length of path or the distribution of the obscuration. Before the introduction of the nebule there was current no unit for the quantity of obscuration, irrespective of any other physical properties of the obscuration, the nature and diversity of its constituents. It is relatively easy to visualize a nebule; it conveys the idea of materiality and facilitates the expression in speech or writing of the relevant facts. Ten nebules mean ten times as much obscuring matter as one nebule; ten extinction coefficients is a meaningless expression. An extinction coefficient looks and sounds like a mathematical abstraction and is not readily visualized – its dimensions are given as L^{-1}. It can be regarded as obscuring matter per unit distance of the light-path, thus bearing a relation to the nebule somewhat similar to that which density bears to mass. As mentioned, the nebule is easy to comprehend; 100 nebules reduce the light transmitted to 1/1,000th of its intensity; it is also a good practical unit for atmospheric obscurity. If the daylight visibility is 10 yd the number of nebules between observer and object is about 105; for visibilities of 100, 1,000, 10,000 yd the numbers of nebules between observer and objects are approximately 85, 67, and 53 respectively. Light transmitted through a medium in which the extinction coefficient is uniform, and equal to unity would be reduced by nearly 2/3rds in unit distance, 1 cm in the CGS system and 1 m in the MKS system, 1 ft in the FPS system. The atmosphere is practically never so obscure as this; the unit is too large for practical purposes and, as it depends on the unit of length, it may easily be ambiguous. The nebule is independent of the system of units; possibly if it had been chosen so that ten nebules reduced the light to $1/e$ of its value (and 69 nebules transmitted 1/1,000th of the incident light), Middleton might have looked more kindly on it, but it would then have been less easily comprehended by the non-mathematical mind and of a size not so good for the atmospheric range.

The chapter on the extinction of light is a comprehensive presentation of the theoretical treatment and the experimental results of the reflection, refraction, diffraction and absorption of light by spherical drops of diameters varying from a small fraction of the wavelength of light, such as molecules or the smaller nuclei, up to drops with diameters 10 or 20 times the wavelength. Special emphasis is laid on the work of the last twenty years which has added so much to our knowledge of the ' suspensoids ' in the atmosphere and the sizes and number of water drops in fog and cloud. In this connexion it is pleasant to see full acknowledgment of the contribution of H. L. Wright and ' the excellent Kew observations ' and a highly appreciative summary of the ' spider's-web ' investigations of H. Dessens with reproductions of two of Dessens' remarkable photographs of minute haze particles, some of diameter only 1 or 2 μ.

Middleton deduces, via both Mie and Bricard, that small drops, fog drops, of radius 10 μ or more, scatter twice the amount of light that would be expected i.e. the amount scattered by a drop of radius a, is not $\pi a^2 I$ as usually assumed, but $2\pi a^2 I$ where I is the intensity of the incident light. He concludes that the extinction coefficient in a fog with N drops per unit volume is $2\pi N a^2$ or 1·5 m/a where m is the mass of water in unit volume. If, in CGS units, m is 10^{-6} g and a is 2×10^{-3} cm we have Hewson's case of a moderate fog. In a distance of 200 m the opacity would be 217 nebules and the transmitted light would be approximately 3×10^{-7} of the incident light. The daylight visibility would be about 60 m and a 10 candlepower lamp would be just visible in darkness at just over 100 m. These values do not seem unreasonable; incidentally they serve to indicate that practically all of the 38 per cent solar radiation, which Hewson found would be transmitted through 200 m of such a fog or cloud, would be scattered radiation.

There is a very succinct proof that the absorption by water drops is negligible compared with the scattering; the ratio is about 10^{-5} for the drops in a fog; for the largest raindrops of 5 mm diameter the ratio is about 1/400. In rain even the scattering is usually slight and, strangely enough, Middleton repeats ' a curious and unexpected ' (but erroneous) conclusion of F. W. Preston that ' the obscuring power of falling rain is proportional only to the number of drops falling on unit area of the earth's surface per second.' His reasoning, condensed, is as follows, Take drops of radius a, number/cm^3, N, terminal velocity, v. Number X on unit area/sec is Nv. The obscuring power is $2N\pi a^2$ or $2X\pi a^2/v$. By Stokes's law $v = Ca^2$ where C is, effectively, constant. Hence the obscuring power is $2\pi X/C$ and depends only on X. The error lies in assuming that Stokes's law applies to raindrops whereas for these the law gives values far too big; twice the correct value for drops of radius 1/10 mm, and nearly 8 and 20 times the correct values for radii of $\frac{1}{2}$ and 1 mm. For the really large drops of thunderstorms, the ' law ' value is about 60 times too big. Hence for such rain the obscuring power may be substantially greater than in ordinary rain with a predominance of small drops, even though the number of large drops per second is, as it usually will be, much smaller.

Middleton is a little caustic about the suggestion by Sutcliffe that the term fog may cover any type of obscurity which reduces the visibility to less than 1,100 yd – an accidental unjustifiable conclusion from (a) the convention that a fog is counted as a fog if, and only if, the visibility in it is less than 1,100 yd and (b) the fact that meteorological statistics record the number of observations of visibility less than 1,100 yd. Naturally heavy rain or snow or a sandstorm is not fog, even if it reduces the visibility to less than 1,100 yd. Middleton states his own definition ' aerosols containing a large number of water droplets of radius greater than, say, 2 microns.' That definition would let in rain. The international definition is better but it does not seem to have got into observer's handbooks yet though it was approved in 1937. It reads ' Almost microscopically-small water drops suspended in the atmosphere, reducing (according to international convention) the horizontal range of visibility to less than 1 km.' In the supplementary note to the definition, ' The fog generally looks whitish except in the vicinity of industrial regions, where it may be mixed with smoke or fine dust and thus be of a dirty yellow or grey colour.'

In the course of his discussion on the dependence of the extinction coefficient on the wavelength of the light Middleton mentions two questions which he 'has had to parry for years,' viz. what should be the colour of a fog-lamp for night driving and can one photograph through fog with infra-red light ? He continues to parry them but the broad conclusion from the results he has assembled seems to be that in a 'water' fog there is very little difference, if the drops are more than 2 μ radius, but for smaller drops or smoke haze the extinction is appreciably less for red light, and for 'dirty fogs' less for yellow light (in the Fig. 3.9b on p. 41 the curve for 'red' has been printed according to the specification for 'blue').

The visibility of objects depends on contrast and contrast depends on the brightness of the object, B, and the brightness of the background, G. The definition of contrast adopted by Middleton is $C = (B-G)/G$. This does not seem a good definition. If the background is black, the value of C is infinite but the contrast is not. If B and G decrease simultaneously, as by the interposition of a screen, the value of C remains constant but the perceived contrast does not. The threshold value of C, i.e. the value at which the object is just visible, is not constant but increases as the brightness itself decreases. Indeed for a small object the threshold value of C may increase more than one thousandfold as the brightness diminishes.

The significance of this variation of threshold is not always realized, particularly in connexion with observation of objects, as distinguished from lights, with a visibility meter dependent on an optical wedge or dark screens. Wright, in a carefully guarded statement, remarked that 'a black object against a sky background ought to disappear when viewed, under conditions for which the threshold of contrast is 0·02, through a screen of 57 nebules.' But when Poulter questioned this on the basis of observed fact, Wright attributed the difference to increased sensitivity of the eye; this does not seem to be the right explanation. For a near dark object against a sky background, the value of C is, numerically, nearly 1. If a screen is interposed the value of C is still the same and if the threshold remained 0·02, the object would become invisible only when the screen was so thick that both object and sky were blacked out. Actually however, as Middleton's Figs. 5.2 and 5.3 so clearly show, the threshold value increases as the brightness is diminished and, for an object whose threshold is 0·02 for viewing direct against ordinary horizon sky, the threshold becomes 1 when the brightness has been decreased to about 10^{-3} candles/m^2. Then the object disappears (though a much larger object would still be visible against the same sky background). An approximate value for the brightness of the horizon sky on a cloudy day, from Middleton's Table 5.2 is 10^3 candles/m^2. This is reduced to 10^{-3} candles/m^2 by a screen of 200 nebules which is about the value observed and quoted (*Quarterly Journal*, p. 146, 1939).

A better definition of contrast would be one which gave no infinite or negative values and which kept the threshold value approximately constant, whatever the brightness, for an object of a standard shape and apparent size i.e. subtending the same solid angle at the eye of the observer. It would be a function, $f(B, G)$ with a nearly constant value for values of B or G of the order of 10 candles/m^2 or more, decreasing to 1/100 of this constant value for B and G of the order of 10^{-4} candles/m^2. No simple formula will represent exactly the experimental results but $(G - B)/(G + g)$, with $g = 2 \times 10^{-2}$ and G always the greater of the brightnesses of object and background, would avoid the infinite and negative values of contrast and make the threshold value of about the same order of magnitude through the range of brightness from 10^4 to 10^{-4} candles/m^2.

A complete definition would require a function, $f(B, G, O)$ where O is the solid angle subtended by objects of standard shape but of different apparent size. That is for the future (or may be); the existing definition being what it is no one can complain of the adequacy of Middleton's treatment.

In the chapter on visual range in natural light there are nine interesting nomograms giving the ranges of circular targets of different sizes, brighter than the background, viewed against the sky at sky-luminance varying by powers of 10 from 10^3 to 10^{-5} foot-lamberts (it may be necessary but it is a little confusing to have brightness in lumens/cm^2 in chapter three, in candles/m^2 in chap. five and in foot-lamberts in chap. six. The first of these units is about 3,000 times the second and 929 times the third. Middleton gives the ratio of the third unit to the second as 3·42; is it not 3·38 ?) One result immediately apparent from the nomograms is that a very large object about 10 times as bright as the background sky, when the meteorological visibility, $57/n$ or $3{\cdot}912/\sigma$, is 10,000 yd, can be seen up to 20,000 yd in daylight, without much variation from full daylight of a clear day down to twilight, but thereafter a rapid fall to 10,000 yd for starlight and 8,000 yd for 'overcast starlight.'

A formula is given relating the increase in visual range achieved by the use of a telescope and the magnification of the telescope. According to the formula a magnification of 14 would double the visual range and a magnification of 150 would treble it in the case of an object small enough, or contrast low enough, for the naked-eye visual range to be the meteorological range. Middleton points out 'how useless additional magnifying power becomes in thick weather.' The departure of actual telescopes from perfection (assumed in deriving the formula) would reduce the distances by 5 or 10 per cent; but they seem higher than would be expected. In Fig. 6.15, based on the formula, the upper part appears to give a curve of increase in the visual range of a small object, *visible to the naked-eye at a distance of* 10 *km*, in an atmosphere in which the meteorological range is 0·5 km; and in Fig. 7.1, nomogram for the visual range of lights, the intensity scale appears to make a discontinuous jump from 0·9 to 10 candlepower, owing to the omission of 10^0 and the numbers 2 to 9. According to this diagram a light of 0·1 candlepower can be seen against a dark background at a distance of 15,000 yd if the atmosphere is very clear, obscurity about $\frac{1}{4}$ nebule per km. This implies a threshold value of about 4×10^{-10} candles/m^2 or just over 10^{-13} lumens/cm^2 which is about one third of the value found by Luckiesh and Moss, over 20 years ago. Are our eyes three times as good as they used to be ? Or is it just the difference between 'eyes front' and 'eyes right'; foveal and parafoveal vision ?

Middleton mentions a ' happy circumstance,' of consoling interest to those meteorologists who, many years ago, arranged for observations of visibility on the assumption that a sky background of the objects viewed was not essential; it now appears that provided the background is appreciably further away than the object, even if it is a black background, the visibility is practically the same as for a sky background. The proof, as Middleton handsomely acknowledges, is due to F. J. Scrase. For a background 1·5 times as far away as the object, the difference of the visual range from that with a sky background is less than 4 per cent.

In a chapter of 40 pages on ' Measuring visual range ' the various instruments for measuring visibility and scattering are concisely described. Their number is becoming legion. They include instruments depending on photoelectric cells as well as ' eye ' instruments. Most of them are too elaborate for general use at meteorological stations. This is no doubt partly responsible for the result deplored by Middleton that, in the countries of which he had knowledge, no instrument is in widespread use at meteorological stations. So far as daylight observations are concerned, there is much to be said for direct eye observations of suitable objects which can, in fact, usually be found up to substantial distances at most meteorological stations; but it is clearly desirable to have comparable records of the obscuring power of the atmosphere for those times, very frequent times at most places, when it is less than 5 nebules/km. It is also desirable to have some precise measurements of the effective number of nebules/km due to the variations in refractive index arising from small-scale turbulence in the surface layers, i.e. excluding the nebules due to particulate matter; this would need an instrument of very high sensitivity.

Records of night visibility depend on the observation of lights. Middleton dismisses the simple, handy, optical wedge instrument used at stations of the Meteorological Office on the ground that he ' tried it in 1930 but abandoned it as insufficiently precise.' This trial was about 10 years before the instrument was made ! Most of the sources of error in the direct use of a simple optical wedge are in fact eliminated in the Meteorological Office instrument, called the Gold Visibility Meter and in the arrangements for its use, e.g., a large rubber eyeshield to shut out extraneous light from the eye and a long tube to exclude all lights but the ' visibility light '; individual calibration; observation from indoors. With these precautions, the instrument appears to have that moderate precision of Middleton's dictum ' even an instrument of moderate precision would be better than visual observations of lights at night.' He describes graphically and comments caustically on the practical application ' at most airports at least in North America ' of the method of visual observation of lights at night. The use of the Meteorological Office instrument does give comparable values of visibility, very precise for distances between the ' visibility lights ' and reasonably precise for distances up to three times that of the furthest light used. The use of the instrument with objects in daylight depends on the relation between the threshold value and the brightness. An approximate equation for determining the meteorological range V from the number of nebules required to bring two similar objects, at distances r and R, to the limit of visibility, is

$$V = 80\,(R - r)/(m - n)$$

where m and n are the nebules for r and R respectively and R is the greater distance. The relation between brightness, B, and threshold value, E used in the derivation of the equation is, from Middleton's Fig. 5.2

$$\log B = -C - 2 \log E$$

which holds for very small values of B. The constant C depends on the size of the object.

In the last chapters Middleton criticises the international code for reports of visibility, remarking that ' the code is made up of several arithmetic progressions whereas the entire theoretical background of the subject leads naturally and logically to a geometric progression.' How welcome would have been his help at Toronto in 1947 when the British members of the responsible international commission were fighting almost a lone battle for a natural and practical scale of geometric progression, approved a year before at Paris, and then rejected largely through a senseless agitation against a ' logarithmic ' scale and partly, as Middleton surmises, in deference to North American practice. There was not much vision in the atmosphere there.

Naturally, in a book of this kind one notices occasional slips in diagrams or mathematics, e.g. in Fig. 3.5, di_2 is made to represent the angle $i_2 - di_2$; on p. 140 in (7.7) and (7.8) the exp term should be

$$\sigma h\,(r\psi + h)/2r$$

If I have dwelt on some aspects in which I think the book could be improved or amplified I must, to prevent misapprehension, express clearly my admiration for its comprehensiveness and sterling qualities and for Middleton's unremitting efforts to help meteorologists in their task of establishing their records of visibility on a sound basis. In particular this book should be of substantial assistance to the members of the World Meteorological Organization who have to provide internationally comprehensible immediate information of the existing visibility to those who need it in international transport and, at the same time, give the meterologist such information about the distribution of obscuring matter in the atmosphere as will best assist him in meeting the demands of those who need his advice as to future or past visibility.

E. Gold

Tropical Meteorology. By Herbert Riehl. New York (McGraw-Hill) 1954. Pp. x, 392; 253 Figs.; tables. 61s.

This is a timely publication performing the functions of surveying the research work in this field and explaining the importance of the meteorological conditions in the tropics for a proper understanding of the general circulation. The observations in these regions during and since the war have shown that regular diurnal and seasonal variations of weather are not constant features; analysis of the various low-latitude disturbances forms an appreciable part of the discussion.

The book opens with a general view of the wind regimes at the surface and in the upper air, and the distributions of temperature and rainfall in the tropical regions. Evidence revealing a departure from the classical general-circulation theory is obtained from daily and mean monthly contour charts – these do not show a general antitrade as a complement to the low-level trade winds; disturbances exist everywhere in the atmosphere in tropical as well as in higher latitudes. An interpretation of the observed temperature and moisture distributions is given in terms of the dynamic features of the wind field instead of using the idea of air-mass source regions. The important role played by the trade-wind inversion and its effects on the climate of various regions are described.

Analysis of the variability of rainfall at a number of stations shows that most of the rainfall occurs in a few short spells; within relatively few and narrow zones of disturbed weather the tropical atmosphere obtains the largest share of that portion of its heat which is derived from condensation and precipitation.

Chapters follow on ' Diurnal and local effects ' and ' Convection '; the latter opens with a discussion of the energy exchange near the sea surface and suggests that this is important not only for the production of cumulus clouds but is a factor in large-scale and longer-period circulation changes. The limitations of the ' parcel ' and ' slice ' methods in explaining the features of cumulus clouds lead to the theory that environmental air is entrained and mixed with the ascending cloud air.

The ' Physics of tropical rain ' is dealt with by Dr. R. Wexler. A discussion is given of the significant phenomenon of precipitation from clouds with temperatures entirely above freezing point, involving recent theories on the growth of cloud drops by coalescence. The processes of precipitation growth from ice crystals and Byers' work on thunderstorms lead to recent work on artificial precipitation.

Chapter 7 directly concerns the work of forecasters. Special difficulties, e.g., the dominance of the diurnal cycle, are discussed and methods of analysis suitable for the tropics are described in detail such as time sections, streamline analysis and the use of isogons. In Chapter 8, a simplified mathematical treatment is given of the concepts of divergence and vorticity, used in the last three chapters of the book.

Typical conditions of wind, weather, pressure, temperature and humidity in waves in the easterlies are illustrated by observations in the Caribbean and a dynamical theory of these waves is given, using the theorem of conservation of potential vorticity; a discussion follows on the formation and forecasting of the waves and Palmer's treatment of waves in the equatorial easterlies.

In the chapter on ' Low-latitude disturbances ' the author, in agreement with Palmer's work, rejects the air-mass and frontal approach and uses the term ' equatorial trough ' in preference to ' front ' or ' convergence zone.' Three types of disturbances are analysed (a) equatorial trough over the Caribbean, (b) the jet stream in the central tropical Pacific and (c) the Indian monsoon. The second example leads to a comparison of upper-air features in high and low latitudes; the advance of the Indian monsoon is shown to be linked with the westerly displacement of the trough at 500 mb. Finally the interaction between high and low latitudes is illustrated by the work on the ' zonal index.'

The longest chapter is devoted to ' Tropical storms,' a subject whose practical and theoretical importance has attracted many research workers. A description is given of the atmospheric machinery, i.e. the processes by which the energy-input (mainly the latent energy of water vapour) is finally converted into the kinetic energy of winds. Various theories of the formation of tropical storms are discussed and details of the surface and upper-air structure are given; the use of radar showed that the central storm area is not filled with cloud uniformly throughout the troposphere but contains narrow elongated bands of cloud spiralling inwards. A final section on the motion of tropical storms leads to forecasting applications.

The last chapter explains how the various features, previously discussed, fit in with modern ideas on the general circulation. An analysis is given of the heat balance and the momentum balance of the earth-atmosphere system; this analysis is applied to a section of the north-eastern Pacific Ocean. Finally there is a note on the the theory of the formation and the dissolution of the trade inversion.

This work will probably be a standard text on the subject for some time to come. It is handsomely printed and produced, conforming to the usual high standards of the publishers.

J. Pepper.

The properties of ordinary water substance. Compiled by N. E. Dorsey, New York (Reinhold), 1940 (reprinted 1953). Pp. xxiv, 673; 12 Figs., 289 tables. 120s.

This is a second printing (1953) of the 1940 edition of one of the *American Chemical Society Series of Scientific and Technologic Monographs.* It gives information regarding the properties of pure, ordinary water substance in all its phases, – water vapour, water, and the several ices, both in carefully purified forms and also in the more ordinary forms that are found in nature including sea water. In general no information is given regarding its behaviour in the presence of another substance, but the exceptions to this rule are just those that would interest a meteorologist, a glaciologist or an oceanographer. These are that the effects

of presence of air are considered as also are the solubilities and diffusivities in water of the atmospheric and noble gases, of hydrogen, ozone, carbon monoxide, and ammonia, and the diffusion of water vapour in air, and through a few solids. Also some few types of information that might not be expected have been included, for example, the preparation of dust-free water, and of monocrystals of ice, the colour of water and of the sea. The book is no longer new – its period is best given in the author's own words ' It is hoped that no important article bearing on the subject appearing before 1938 has been overlooked; but only the most important of those appearing after 30 June 1937, and coming to the compiler's attention have been considered.'

The whole monograph with 673 pages of close type, 289 tables and at least 2,000 references is a monument to Mr. Dorsey's thoroughness and to the importance and interest of just one chemical substance. The style is clear and highly factual, but not too impersonal, which makes it pleasant to read – a rather rare thing in a reference work of this kind. The method of referencing is good, for it minimises page turning, and the indexing is as thorough as the book. For a general reference work on water substance it is to be recommended, though its very wide scope makes it more suited to the bookshelves of a library than of an individual who would normally use only a small part of it.

The date (1938) of latest information in it, is in some sections rather a handicap (in some of course it is none). In respect of the question of the freezing of supercooled water, a subject in which both the author and the present reviewer happen to be interested, it is clear how far we have progressed since 1938. This would also be true of the section on the diffusion of vapour and liquid water through other substances.

A.W.B.

Klimatologie der Nordwesteuropäischen Gewässer, Parts 1 and 2. Deutscher Wetterdienst, Seewetteramt, Hamburg, 1954. Einzelveröffentlichungen No. 4. Pp. 59; 111 charts, 4 tables. 7 D.M.

This publication was prepared in the Seewatteramt of the Deutscher Wetterdienst under the overall direction of Prof. Kuhlbrodt. Part 1 contains various functions of the sea surface temperature and the air-minus-sea temperature difference while Part 2 contains wind information. Part 3 is to be published separately and will contain information on the distributions of cloud, visibility and precipitation as well as that of air temperature. The ships' observations on which the charts are based are mainly for the years 1906-14 and 1920-39.

Part 1, which was prepared by Bullig and Bintig, includes some charts which are an innovation in such atlases. These charts showing the change of mean sea temperature between the period 1906-13 and 1922-29 for the four seasons for selected areas are a feature which will be appreciated, and it is to be hoped will be followed by further efforts to depict the climatic variations in such atlases, as well as the means for all the period for which observations are available.

The sea-temperature monthly means have been computed for one-degree squares, reliable values being possible since the area is one where ships' meteorological observations are plentiful. The actual values of the means for each square are shown as well as the isotherms. The annual variation of sea surface temperature is shown, diagrammatically, for eighteen chosen areas.

The maximum and minimum sea-temperature values shown are the five-percentiles, a procedure which was also used in the preparation of the British marine meteorological atlases. Isopleths of the range of sea temperature (the interval between the five-percentile maximum and minimum temperature) are shown for four representative months, together with the standard deviation, the annual variation of which is also shown. Charts of the air-sea temperature difference are given for the four seasons, together with the annual variation for selected areas.

Part 2 was prepared by Markgraf and Bintig and commences with monthly charts of the mean of the Beaufort wind forces for two-degree squares followed by the representation of the frequencies of groups of Beaufort forces for each month for eighteen chosen areas : the annual variation of the frequencies is then shown graphically. Monthly charts of wind roses are then shown and these are followed by charts for four months of the year of the predominant wind direction and the direction of the predominant sector of winds for nineteen points : a chart of the annual variation of predominant wind direction for the same nineteen points comes next.

The charts of mean monthly zonal and meridional components follow, and constitute a valuable addition to this type of atlas for the meteorologist, although for the mariner they are likely to be of little use. Then come charts of the direction and speed of the mean vector wind for four months. There follow charts for four months of the constancy of the wind, defined here as the ratio of the mean vector wind-force to the mean scalar wind-force, expressed in tenths. To complete Part 2 the annual variation of the vector wind and the percentage constancies are given for eighteen areas.

This atlas, like many others, shows the difference between the definitions of a few terms, e.g. predominant direction and constancy of winds, and the very different modes of representation of various elements in use in various countries. Means of achieving some uniformity in the production of such atlases have been proposed in the World Meteorological Organization.

The atlas is well produced, the printing being very good, and the price is reasonable. It will form a useful addition to the publications on the meteorology of this area.

P.R.B.

Indian Ocean – Oceanographic and Meteorological Data. Koninklijk Nederlands Meteorologisch Instituut. Second Edn., 1952, No. 135. Fl.61.50.

This publication consists of twenty-four separate sheets, two for each month, and a separate booklet containing the text.

On sheet A for each month the following information is given on a main chart : (a) Surface ocean-current roses for specified areas shown on the chart. (b) The direction and velocity of the mean vector currents for two-degree squares. The constancy of the current is defined as the ratio of the mean vector velocity to the scalar mean and is shown within ranges by the thickness and length of the arrow giving the direction. (c) Isotherms of mean air temperature. (d) Isotherms of mean sea temperature. On one smaller side chart on sheet A are shown the surface current-circulation, based on the computed predominant currents, and the standard deviation of sea-surface temperature-observations, while on another are shown the mean cloud amount, the percentage of fog duration, the duration percentage of precipitation and the standard deviation of air-temperature observations. Also on sheet A are shown current drifts of 97 mi/day or over observed during the month.

On chart B the following information is given : (a) Wind roses for five-degree squares. (b) The direction and velocity of the mean vector winds for two-degree squares shown as feathered arrows, which also denote by length and breadth the constancy, defined as for surface currents. (c) The mean isobars. (d) The percentage frequency of gales (Beaufort force 8 or more). (e) The number of tropical cyclones observed in each five-degree square. On the side charts of sheet B are shown the tracks of tropical cyclones and the years in which they were observed.

The text, apart from explaining the mode of preparation of the statistics shown on the charts, contains a very fine account of the occurrence of tropical cyclones in the Indian Ocean.

While the many colours used in the printing help in picking out the various elements on the charts, it would have been preferable for clarity, at least from the meteorologists' viewpoint, to present the information on more numerous separate charts. On the other hand many mariners prefer to have the vital information on as few charts as possible for easy reference. These conflicting requirements of the meteorologist and mariner can only be solved satisfactorily if a country is in a position to publish both meteorological atlases and separate charts. The publishing of one or two such charts for each month necessarily means the omission of some information, in this instance that on swell, lightning and air-sea temperature difference. Standard deviations of air and sea temperature are given and will undoubtedly be of great value, but one cannot help feeling that many mariners, shipowners, shipbuilders and others would find maximum and minimum temperatures and ranges of temperature more convenient.

These few criticisms are not meant to detract from the value of these charts, which are the most up-to-date ones for the Indian Ocean. Anyone concerned with marine meteorology in this area will wish to have them for reference and mariners might well find them, considering the information they contain, the most convenient yet produced. The printing, as for other Dutch meteorological charts, is really excellent.

P.R.B.

Climatological Atlas of Canada. Pp. 253; 84 maps. $2.

The Climatological Atlas of Canada was primarily produced for use in the design of buildings and is published jointly by the Division of Building Research of the National Research Council of Canada and the Meteorological Division, Department of Transport.

The larger part of the Atlas consists of 76 charts of isopleths of temperature, humidity, wind, snow, rain, sunshine, earthquake probability and permanently frozen soil. They cover Canada and the islands of Arctic Canada on the scale of about $1:28 \times 10^6$. In addition, there are hythergraphs, that is diagrams in which mean monthly temperature as ordinate is plotted against mean monthly precipitation as abscissa, for 24 well distributed stations over Canada and for Washington D.C., London (Kew) and Paris, and also tables of solar elevation.

It is not possible to give a detailed description of this large work in the available space and comment must be confined to the more interesting features.

Much information on means and extremes of temperature is give with isopleths of 5° or 10°F intervals. The means are not reduced to sea-level values. Over mountainous areas the isopleths are based mainly on observations at valley stations. It is recognized in the preface that the maps can indicate only the broad features of the distribution. The isopleths no doubt give a reasonable representation over the wide plains but over, say, British Columbia they would not suffice for obtaining a local value. The maps include eight of ' design temperatures ' which are, for winter, the temperatures such that hourly temperatures in January are below them for respectively 1, 2½, 5 and 10 per cent of the total time. For summer (July) they are compiled on the converse excess basis. It is interesting to compare the design temperatures with the extremes. The winter design temperature on a 10 per cent basis is 10° to 20°F above the 1 per cent value. The mean January minimum temperature is about 10°F above the 10 per cent design temperature and the mean annual minimum temperature is about equal to the design temperature on a 1 per cent basis. No indication is given about which of the design temperatures is recommended for use in different kinds of building.

The wind maps include maps of expected maximum gusts computed from an empirical linear relation between the maximum gust recorded by a Dines anemometer and the maximum hourly run of wind of a cup anemometer at the same place. There is a wide distribution of cup anemometers over Canada and the

maximum gusts were computed by this formula from their hourly run records. The computed maximum gust is 120 mi/hr over much of the eastern seaboard. In British building practice the maximum gust to be expected over a few seconds is considered high for wind-pressure computations though much depends on the exposure of the building.

The most striking map among those for snow is that of the computed maximum snow load on a horizontal surface obtained by adding to the maximum recorded depth of snow on the ground the estimated maximum amount of rain which might fall into the snow cover in twenty-four hours. This computed load reaches 80 to 90 lb/ft^2 in parts of the Rockies and in Labrador.

General precipitation maps include, besides mean annual and seasonal totals and mean numbers of rain days, one for the maximum amount of rain expected to fall in 15 min once in ten years. The last was based on a formula developed in U.S.A. relating rainfall intensity to the average amount of rainfall in a rainy day. The highest value, 1 in., appears in the extreme south.

Mean monthly maps of bright sunshine are published but there are no actinometric data. A table of the approximate intensity of solar radiation at normal incidence against solar elevation would have been a useful supplement to the solar elevation table for computing radiation incident on a building. No indication of orography is shown on the maps; an additional physical map would have been useful for easy comparison of the distributions with the orography.

Perhaps in future editions it will be possible to extend the contents more beyond the range of those of particular importance in building and include information on visibility, fog, cloudiness, thunderstorms and hail with a few synoptic charts illustrative of the major meteorological situations of Canada.

G.A.B.

Climate and Architecture. J. E. Aronin. New York (Reinhold), 1953. Pp. x, 304; 287 Figs.; 57 tables; index. $12.50.

Like many modern buildings, this book, from the publisher's Progressive Architecture Series, is large, striking, and quite expensive; with bold-type faces, double columns, unusual margins, off-centre illustrations superbly reproduced, and very clear diagrams which sometimes take more space than is necessary. Too large for the domestic bookshelves of the majority of young architects who must live in modern flats, it will look well lying on a table surrounded by fitted carpets in a very new office building. Indeed, this reviewer finds it not a little evocative of the jolly nineteen-thirties when lines were pure, black was black and it was assumed that the intelligentsia did not have to worry about their basic subsistence allowance.

The author, writing for the most part in the cheerfully direct, colloquial and rather staccato style that we are learning to expect from many American texts, has produced an agreeably unpretentious, at times elementary, but interesting and comprehensive assemblage of material. He builds his book largely with the aid of well-chosen extracts, on almost every aspect of climate as it affects the work of the architect. Like many modern buildings, the structure is assembled from prefabricated materials by good manufacturers; and the design is satisfactory in relation to its environment and may well please the clients. After thirty pages of introduction and an effective summary of the historical background, nearly ninety pages under the heading ' the sun ' are devoted to insolation and the many problems of design arising therefrom; not excluding, for example, the planting of shade trees. Sixty pages are concerned with the theme of temperature, fifty with wind. Photographic studies of the airflow around and through buildings, carried out in a wind tunnel at the University of Texas (pp. 202-3) attract attention and there is a useful summary of American work on shelter-belts. Forty pages are devoted to precipitation, including comments on snow-fences (pp. 243-5) and a brief, apparently hurried, section follows on design in relation to other climatic factors such as lightning and humidity. There is a useful and indeed very extensive bibliography in which the expected meteorological references will be found. A few French, German and Swedish papers are included, but no mention is made of Russian sources which might well be appropriate to the American environment.

The author's declared object is ' to acquaint architects, architectural students, engineers, city planners, builders, house owners, especially those in North America, with the demands and phenomena of . . . macro- and microclimate.' The meteorology is elementary, and sometimes needs qualification, e.g. with regard to radiation fog over a snow surface (p. 228). Citations and illustrations of the work of numerous architects, mostly American, abound; Le Corbusier is viewed with appropriate regard.

Yet although it is primarily compiled for architects, meteorological readers can dip in with enjoyment and find much to interest them on the application of numerous simple physical principles. The design of houses (not to speak of books) for the sunny climate of Texas, and for conditions of extreme cold yet strong sun such as are common in the Middle West demands very different attitudes from those customary in North-West Europe. With the aid of quotations and references (among which Rickmansworth appears) the author provides a useful description of various microclimatological phenomena in so far as they affect the architect and engineer. The author is a graduate of McGill, and his personal experience of Montreal, a city presenting a wide variety of climatic difficulties, is well to the fore. Likewise, his comments on early French-Canadian adaptations are welcome.

Like most places of architecture the book is not free from blemish; minor slips exist such as ' data ' used as a singular (p. 237), occasional misprinting, e.g. of Taunus on p. 270 and Grabham on p. 163; some slight confusion arises from the apparent telescoping of sentences, e.g. on Greenwich time (p. 49) and on sleet

and glaze (p. 227); on p. 270 cubic centimetres would appear to be implied – these, however, do not detrac from the book as a whole. The splendid illustrations of some of the successes of modern architecture i creating comfort, or partial comfort, in climates far from our own, and the stimulus to the mind which i bound to arise from visible signs of the use of intelligence in strange environments, will lead some at leas to reconsider emigration. Others will wonder how soon the peculiar problems of a climate such as tha of Glasgow will receive further thought, epitomised as far as North America is concerned by this simpl and cheerful compilation.

G.M.

Soil. By G. V. Jacks. London (Nelson), 1954. Pp. ix, 221; 10 plates, 7 Figs.; 4 tables. 12s. 6d.

So comprehensive a title might well serve for a series of a dozen volumes by as many authors. Mr. Jack collects the salient features of agricultural significance and presents them in a dozen or so chapters for th benefit of those who practise, or aspire to practise, enlightened farming. He covers the usual ground familia to those, for example, who read for ordinary degrees in agriculture in our Universities, but with more than the usual coherence. We find the expected chapters on soil description, soil chemistry (conventionally held to include plant nutrition), and soil water, together with descriptive soil microbiology and an account of th consequences of biological activity. Throughout the book runs the theme that man is a biological agent a natural and as legitimate an ecological factor as any earthworm or micro-organism, and that his function conscious or unconscious, is to stabilize the agriculture by which he supports his civilization by developing a stable soil crumb structure. A purist might find cause for regret in the occasional lapse into teleology.

In so compact a work, repetition is undesirable, very noticeable where it occurs, and very difficult to avoid since the same facts may be adduced in different contexts. Great economy of words here and there leads to some lack of perspicuity and even to apparent self-contradition. Mr. Jacks might have written an even better book by making a little more room for himself. He might have done this by having the full courage of his convictions (he believes that only a little of science is useful in the agricultural study of soil) and omitting some of his exposition of physical science; for the necessarily cursory treatment may contribute little to his audience's understanding.

E.C.C.

Obituary Notices

Major R. McN. KER

A Life Fellow of 34 years' standing was lost to the Society by the sudden death on 15 October 1953 of Major Robert McNeil Ker. Born in 1878, he was educated at Rugby and entered the army, serving in the South African war with the 3rd Battalion, The Argyll and Sutherland Highlanders. During the war of 1914-18 he went to France and to the Salonika front as a major in the Machine Gun Corps. An enthusiastic amateur meteorologist, Major Ker maintained systematic records of weather, rainfall and temperature over a long term of years at Milngavie, Dumbartonshire, his home town. This interest he inherited from the two preceding generations of the family. His father, Thomas Ripley Ker (1854-1947), one of the Life Fellows of the Scottish Meteorological Society who transferred to the Royal Meteorological Society on the amalgamation of the two bodies in 1921, kept a day-to-day log of the weather in his personal diary from 1874 to 1941, Major Ker's grandfather, Robert Ker (1799-1888), began regular meteorological observations at Auchinraith, Hamilton, Lanarkshire in 1853 and was a contributor to the first volumes of G. J. Symons's *British Rainfall*, issued early in the 1860's. We are informed by Major Robert McNeil Ker's only son, Mr. Neil R. Ker, who carries on to the fourth generation the family tradition of interest in meteorology, that he possesses a manuscript record of his great-grandfather, Robert Ker, of pressure, temperature and state of weather as observed by himself at Manila four times daily from July 1831 to January 1832.

E.L.H

Mr. L. G. H. LEE

Leon George Harold Lee, who died suddenly at the age of 71, at his home at Raunds, Northamptonshire on 6 July 1954, had been a Fellow of the Society for 50 years, having been elected on 18 May 1904. Lee was the son of the headmaster of the Wesleyan School at Raunds and it was therefore natural that he should complete his formal education at Westminster Training College. In 1906 he followed his father as headmaster of the School at Raunds. He also took charge of the meteorological station there : one of the stations whose observations were at that time collected by the Society and published in the *Record*. From October 1907, the Meteorological Office, by arrangement with the Society, began to publish summaries from a number of these stations, including Raunds, in the *Monthly Weather Report*. Lee agreed also to send the Meteorological Office a weekly report and his station became a ' District Value ' station for both the weekly and the *Monthly Weather Report*. It was therefore natural, when meteorological observers were required in the summer of 1915 by the new Meteorological Service with the Army in France, that Lee's transfer, from the Northamptonshire Regiment in which he was then serving in the rank of Sergeant, should be sought. It was, in fact, speedily effected. He was transferred to the Royal Engineers on 14 July and arrived in France on 23 July 1915. He was immediately stationed at an observation post at Nieppe, an unhealthy place, near Armentières, and was later moved to Etinehem on the Somme, about 20 miles east of Amiens. At first he was single-handed, making reports to Headquarters at 1 a.m., 7 a.m., 1 p.m., 6 p.m. and at intermediate times as occasion required. Later he had a succession of assistants, sent partly to gain from him some of his special knowledge of the art of making worthwhile meteorological observations, both by day and by night, just behind the front line. He had no light task. The following is the last (and only surviving) of some verses in which Lee described, with a touch of his kindly humour, the lot of ' The Meteorological Observer ' (in France in the first World War),

Toiling, rejoicing, sorrowing,
Onward through life he goes.
One a.m. sees his work begun
And midnight sees it close.
Something attempted
And a good deal guessed
Has earned an hour's repose.

Lee was mentioned in Despatches in 1917 and 1918. He was demobilised in March 1919.

On his return to civil life, he entered at Gray's Inn and was called to the Bar in 1922. A year later he was appointed one of H.M. Inspectors of Schools : in Lincolnshire, Gloucestershire, Cheshire and Lancashire at different times in his period of service. After his retirement, in 1944, he became a member of the Northamptonshire County Council, Chairman of the Education Committee and, in 1953, County Alderman.

Lee was sturdy in physique and in character, a lover of the beauties of nature, in the sky and on the earth. He originated the expressive term ' weak sky ' for a certain developing cloud system indicative of approaching rain.

Outstanding in the recollections of his service in France, which he emphasized on more than one occasion when he came in the 1920's to the reunion dinners of the Meteorological Section, was the solemn beauty of the early dawn of those summer mornings on the Somme. The DAWN, he would say, in a tone which did convey something of the inexpressible feeling of awe and inspiration in the presence of that scene.

But at Headquarters it was the trustworthiness of his reports (which somehow or other he always manage to get through) that remained as an abiding memory.

In preparing this notice the writer has had the advantage of an outline of Lee's career kindly supplie by Mr. T. C. Smith, his friend and his successor in maintaining the meteorological station at Raunds; an is indebted to Mr. J. Durward for his preservation of the verse.

E.G.

Wing Commander D. M. EGAN, A.F.C.

Dermot Mitchell Egan, who had held Fellowship in the Society since 1947, died on 13 January 1954, a few days before the close of his 46th year. He was educated in Dublin, joined the Royal Air Force as an aircraft apprentice at the age of 17, qualified as a pilot in 1932, and after a term of service in the Middl East with No. 216 Squadron became a flying instructor in 1937. He received the King's Commendation for brave conduct on an occasion in 1941 when during a night flight his aircraft was shot down in flame and his pupil killed. In 1942 he was awarded the Air Force Cross. After a period of staff duties Win Commander Egan was appointed Chief Flying Instructor at No. 1 Glider Training School; he was the posted to Headquarters Transport Command as Command Air Traffic Officer. Subsequently he serve for five years at the Air Ministry. Sailing was among his chief recreations, and the study of meteorology had long attracted him in its bearing on this as well as on his chosen profession. For details of Win Commander Egan's career we are indebted to the courtesy of his friend Wing Commander B. E. Crawley Headquarters Coastal Command, Royal Air Force.

E.L.H.

Corrigendum to October 1954 issue

G. A. Corby, p. 496, lines 24 and 25—

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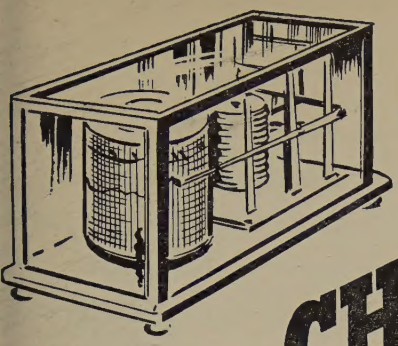

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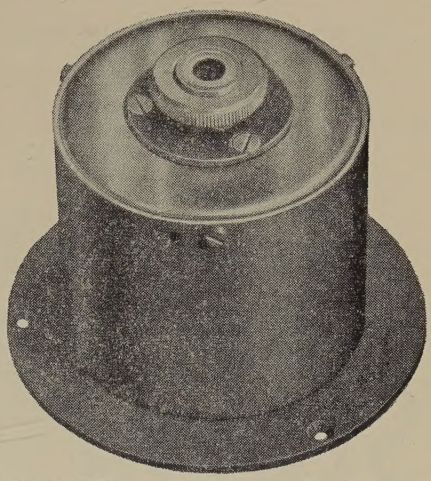

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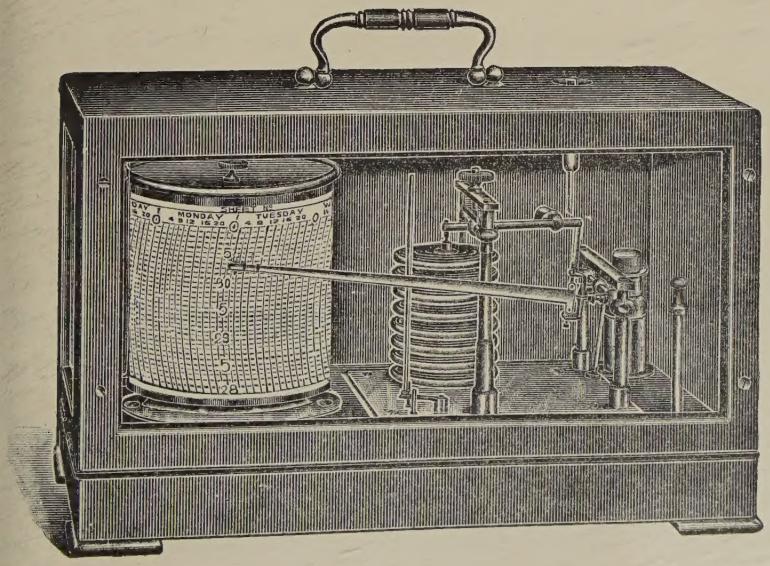

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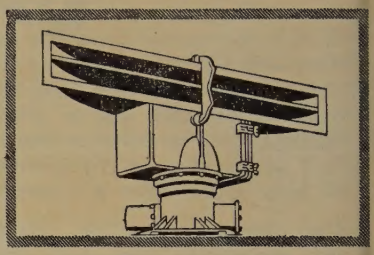

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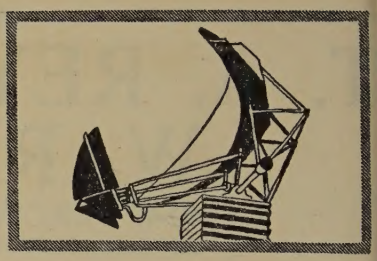

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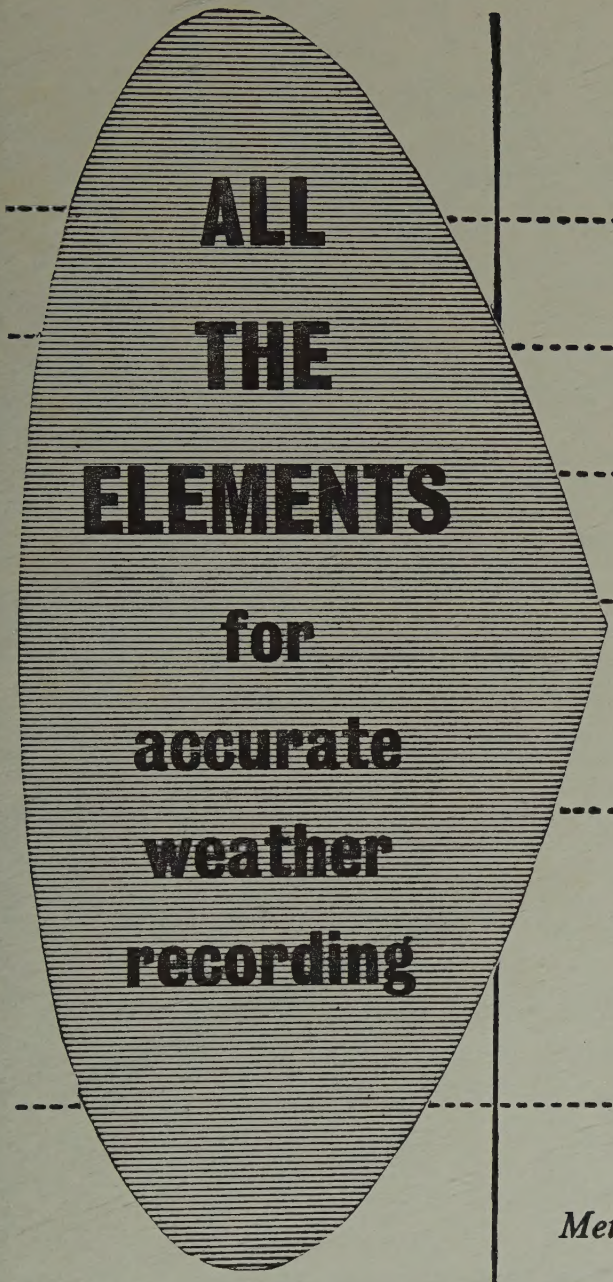

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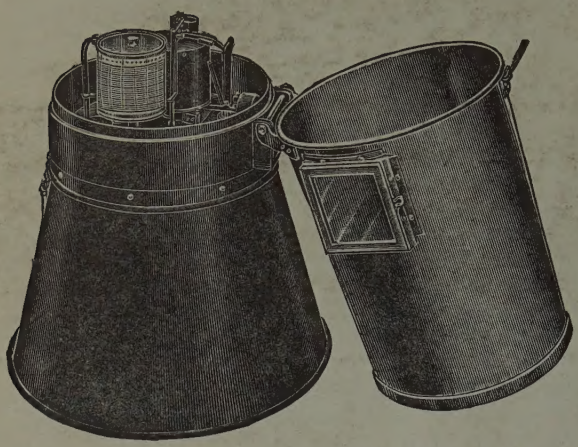

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